Solucionario Trigonometría 2.°

ÁNGULO TRIGONOMÉTRICO Y SISTEMAS DE MEDIDAS ANGULARES

APLICAMOS LO APRENDIDO (página 6) Unidad 1

1. $E = \dfrac{\frac{\pi}{3}\text{ rad} + \frac{\pi}{4}\text{ rad} + 36°}{20^g + 30^g + \frac{\pi}{5}\text{ rad} + 50^g}$

$E = \dfrac{\frac{\pi}{3}\text{ rad} \times \frac{200^g}{\pi\text{ rad}} + \frac{\pi}{4}\text{ rad} \times \frac{200^g}{\pi\text{ rad}} + 36° \times \frac{10^g}{9°}}{50^g + \frac{\pi}{5}\text{ rad} \times \frac{200^g}{\pi\text{ rad}} + 50^g}$

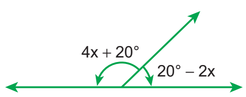
$E = \dfrac{\frac{200^g}{3} + 50^g + 40^g}{50^g + 40^g + 50^g}$

$E = \dfrac{\frac{200^g + 270^g}{3}}{140^g}$

$E = \dfrac{470^g}{3 \times 140^g} \Rightarrow E = \dfrac{47}{42}$

Clave D

2.



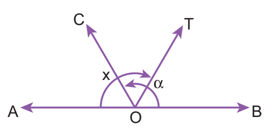
$4x + 20° - 20° + 2x = 180°$

$6x = 180°$

$x = 30°$

Clave C

3. Del gráfico:



Dato $\overrightarrow{OT}$ es bisectriz del $\angle BOC$, entonces:

$m\angle COT = m\angle TOB$

Ahora:

$-x + \dfrac{\alpha}{2} = 180°$

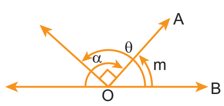
$-x = 180° - \dfrac{\alpha}{2}$

$x = \dfrac{\alpha}{2} - 180°$

$x = \dfrac{\alpha}{2} - \pi$

Clave D

4. Del gráfico:



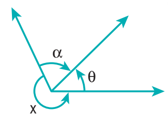
Sea el $\angle AOB = m$

$\left.\begin{aligned} \theta - m &= 90° \\ -\alpha + m &= 180° \end{aligned}\right\}$ Sumando

$\theta - \alpha = 270°$

Clave D

5.



$\theta - \alpha + x = 360°$

$x = 360° - \theta + \alpha$

$x = 2\pi - \theta + \alpha$

Clave B

6. I. $360° > 2\pi$ (F)

II. $1° = 60'$ (V)

III. $9° < (10^g)$ (F)

De I: $360° = 2\pi$ rad

De III: $9° = 10^g$

Clave C

7. $n° + (10n)^g = 90°$

$n° + (10n)^g \times \dfrac{9°}{10^g} = 90°$

$n° + 9°n = 90°$

$10n = 90$

$n = 9 \Rightarrow n° = 9°$

El menor es n°, por lo tanto:

$9° \times \dfrac{\pi}{180°} = \dfrac{\pi}{20}$ rad

Clave C

8. $(10x^2 + x + 4)^g = (9x^2 - x + 20)°$

$(10x^2 + x + 4)^g \cdot \dfrac{9°}{10^g} = (9x^2 - x + 20)°$

$90x^2 + 9x + 36 = 90x^2 - 10x + 200$

$19x = 164$

$x = \dfrac{164}{19}$

Clave D

9. $2C - \dfrac{S}{2} = 31$

$4C - S = 62$

$4(10k) - (9k) = 62$

$31k = 62$

$k = 2$

$\therefore x = 9k = 9(2) = 18° \times \dfrac{\pi}{180°} = \dfrac{\pi}{10}$ rad

Clave A

10. $E = \dfrac{S^2 + C^2 + SC}{SC}$

$E = \dfrac{(9k)^2 + (10k)^2 + (9k)(10k)}{(9k)(10k)}$

$E = \dfrac{81k^2 + 100k^2 + 90k^2}{90k^2}$

$E = \dfrac{271k^2}{90k^2} = \dfrac{271}{90}$

Clave B

11. Se sabe:

$\dfrac{S}{9} = \dfrac{C}{10} = k$ ...(1)

Del dato:

$S = 45°$ y $C = 50^g$

Reemplazando en (1):

$\dfrac{S}{9} = k \Rightarrow \dfrac{45}{9} = 5 = k$

$\dfrac{C}{10} = k \Rightarrow \dfrac{50}{10} = 5 = k$

Piden:

$\dfrac{S + 15}{C - 10} = \dfrac{9k + 15}{10k - 10} = \dfrac{60}{40} = \dfrac{3}{2}$

Clave A

12. $17{,}72° = 17° + 0{,}72° = 17° + \dfrac{72}{100} \times 60'$

$= 17° + \dfrac{432'}{10}$

$17{,}72° = 17° + 43{,}2' = 17° + 43' + 0{,}2'$

$= 17° + 43' + 0{,}2 \times 60''$

$17{,}72° = 17° + 43' + 12''$

$\therefore 17{,}72° = 17°\,43'\,12''$

Clave B

13. Del problema, n número de minutos sexagesimales:

$n = 60S$, S: número de grados sexagesimales

Además de: $\dfrac{S}{9}$; $\dfrac{C}{10}$ se tiene que:

$\dfrac{S}{9} = \dfrac{50}{10} \Rightarrow S = 45$

Luego:

$n = 60(45)$

$n = 2700$; reemplazando en M

$M = \dfrac{\sqrt[3]{2700 \times 10} + 30}{4} = \dfrac{30 + 30}{4}$

$\therefore M = 15$

Clave E

14. $108^g - 108° = 108^g \times \dfrac{9°}{10^g} - 108°$

$= \left(\dfrac{108° \times 9}{10}\right)° - 108°$

$108^g - 108° = 108°\left(\dfrac{9}{10} - 1\right)$

$= 108°\left(\dfrac{-1}{10}\right)$

$108^g - 108° = -10{,}8°$

El error es 10,8°; luego $10{,}8° = 10{,}8° \times \underbrace{\dfrac{\pi\text{rad}}{180°}}_{\text{Factor de conversión}}$

$\therefore 10{,}8° = \dfrac{3\pi}{50}$rad

Clave C

PRACTIQUEMOS

Nivel 1 (página 8) Unidad 1

Comunicación matemática

1. Cuando un ángulo se expresa en subunidades se tiene en cuenta:

Sexagesimal: $a°b'c''$ $\forall$ $\wedge$ $b < 60$, $c < 60$

Centesimal: $x^g\, y^m\, z^s$ $\forall$ $\wedge$ $y < 100$, $z < 100$

Luego se observa:

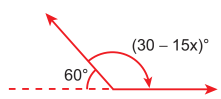
$127^g\, 77^m\, 20^s$, respuesta

Clave C

2.

Razonamiento y demostración

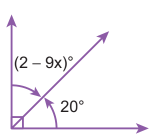
3. Del gráfico:



$$60° - (30 - 15x)° = 180°$$
$$60 - 30 + 15x = 180$$
$$x = 10$$

Clave A

4.



$$20° - (2 - 9x)° = 90°$$
$$20 - 2 + 9x = 90$$
$$9x = 72$$
$$x = 8$$

Clave D

5. Del gráfico:

$30° = -(9 - 3x)°$

$30° = -9 + 3x$

$\Rightarrow x = 13$

Clave E

6. Del gráfico:

$\alpha - \beta + x = 360°$

$x = 360° + \beta - \alpha$

Clave D

7. $M = \sqrt{\dfrac{C+S}{C-S} + \sqrt{\dfrac{C+S}{C-S} + 17}}$

Se sabe:

$S = 9k$ y $C = 10k$

Reemplazando:

$$M = \sqrt{\frac{10k+9k}{10k-9k} + \sqrt{\frac{10k+9k}{10k-9k} + 17}}$$

$$M = \sqrt{19 + \sqrt{19 + 17}}$$
$$M = \sqrt{19 + \sqrt{36}}$$
$$M = \sqrt{19 + 6}$$
$$M = \sqrt{25}$$
$$M = 5$$

Clave E

8. $J = \dfrac{40^g}{\dfrac{\pi}{10}\text{ rad}}$

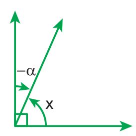
$\dfrac{\pi}{10}$ rad al sistema centesimal:

$$\frac{C}{10} = \frac{20R}{\pi} \Rightarrow \frac{C}{10} = \frac{20}{\pi}\left(\frac{\pi}{10}\right) \Rightarrow C = 20$$

$$\therefore\ J = \frac{40^g}{20^g} = 2$$

Clave B

9.



Ambos ángulos suman 90°:

$x - \alpha = 90°$ $\quad \therefore\ x = 90° + \alpha$

Clave B

Resolución de problemas

10. $\dfrac{\pi}{10}$ rad $+ x = 90°$

$$\frac{\pi}{10}\text{ rad} \times \frac{200^g}{\pi\text{ rad}} + x = 90° \times \frac{10^g}{9°}$$
$$20^g + x = 100^g$$
$$x = 80^g$$

Clave E

11. Si $\overleftrightarrow{OB}$ bisectriz, entonces:

$(21n + 5)^g = (18nb + 27)°$

$(21n + 5)^g \cdot \dfrac{9°}{10^g} = (18n + 27)$

$9(21n + 5) = 10(18n + 27)$

$21n + 5 = 10(2n + 3)$

$21n + 5 = 20n + 30$

$n = 25$

Finalmente:

$$\frac{n+5}{6} = \frac{25+5}{6} = \frac{30}{6} = 5$$

$$\therefore\ \frac{n+5}{6} = 5$$

Clave E

12. $(7x + 1)° = (9x - 5)^g$

$(7x + 1)° \times \dfrac{10^g}{9°} = (9x - 5)^g$

$70x + 10 = 81x - 45$

$-11x = -55$

$x = 5$

Reemplazando y convirtiendo a radianes:

$(7x + 1)° = (7(5) + 1)° = 36°$

Entonces:

$$36° \times \frac{\pi\text{ rad}}{180°} = \frac{\pi}{5}\text{ rad}$$

Clave D

Nivel 2 (página 8) Unidad 1

Comunicación matemática

13. Del gráfico:

$2x\text{ rad} + 100^g + 90° + 90° = 360°$

$2x\text{ rad} \cdot \dfrac{180°}{\pi\text{rad}} + 100^g \cdot \dfrac{180°}{200^g} = 180°$

$x\left(\dfrac{360}{\pi}\right)° + 90° = 180°$

$\dfrac{x360}{\pi} = 90 \qquad \therefore\ x = \dfrac{\pi}{4}$

De las expresiones (I; II; III) se obtiene:

I. Falso

II. Falso

III. $8x\text{ rad} = \dfrac{8\pi}{4}\text{ rad} = 2\pi\text{ rad}$

$\therefore$ m∠1vuelta; verdadero

Clave C

14. En (A)

$100° < 100^g$

$100° < 100^g \cdot \dfrac{9°}{10^g}$

$100° < 90°$

$\therefore$ A es falsa

En (B)

$\dfrac{\pi}{4} + 45° + 20^g = 130^g$

$\dfrac{\pi}{4} \cdot \dfrac{200^g}{\pi} + 45° \cdot \dfrac{10^g}{9} + 20^g = 130^g$

$50^g + 50^g + 20^g = 130^g$

$120^g = 130^g$

$\therefore$ B es falso

En (C)

$\underbrace{\pi\text{ rad}} > 45°$

$\pi\text{ rad} \cdot \dfrac{180°}{\pi\text{rad}} > 45°$

$180° > 45°$

$\therefore$ C es verdadero

En (D)

$3\pi + 180° > 900^g$

$3\pi \cdot \dfrac{200^g}{\pi} + 180° \cdot \dfrac{10^g}{9°} > 900^g$

$600^g + 200^g > 900^g$

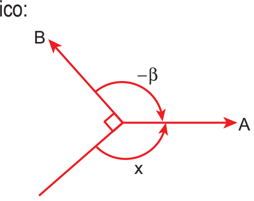
$800^g > 900^g$

$\therefore$ D es falso

Clave C

Razonamiento y demostración

15. Del gráfico:



$$x - \beta + 90° = 360°$$
$$x = 270° + \beta$$

Clave D

16. Del gráfico:

$$30° + 80^g + (360° - \theta) = 180°$$
$$30° + 80^g \times \frac{9°}{10^g} + 360° - \theta = 180°$$
$$30° + 72° + 360° - \theta = 180°$$
$$-\theta = -282°$$
$$\theta = 282°$$

Clave E

17. Del gráfico:

$$(4n + 12)° - (2 - 7n)° = 120°$$
$$4n + 12 - 2 + 7n = 120$$
$$11n = 110$$
$$n = 10$$

Clave B

18. $80^g \times \frac{9°}{10^g} = 72°$

$$\frac{3\pi}{4}\text{rad} \times \frac{180°}{\pi\text{rad}} = 135°$$

Clave A

19. $234° \times \frac{10^g}{9°} = 260^g$

$$\frac{\pi}{5}\text{rad} \times \frac{200^g}{\pi\text{rad}} = 40^g$$

Clave B

20. $37^g \quad 294^m \quad 600^s$

$600^s = 6^m$

$294^m + 6^m = 300^m = 3^g$

$37^g + 3^g = 40^g$

$43° \quad 114' \quad 360''$

$360'' = 6'$

$114' + 6' = 120' = 2°$

$43° + 2° = 45°$

Clave B

21. $\frac{S + C}{38} = \frac{3R^2}{\pi^2}$

Se sabe:

$S = \frac{180R}{\pi} \qquad C = \frac{200R}{\pi}$

Reemplazando:

$$\frac{\frac{180R}{\pi} + \frac{200R}{\pi}}{38} = \frac{3R^2}{\pi^2}$$
$$\frac{380R}{38\pi} = \frac{3R^2}{\pi^2}$$
$$10 = \frac{3R}{\pi}$$
$$\Rightarrow R = \frac{10\pi}{3}$$

Clave C

Resolución de problemas

22. $70^g + 50^g + x = 180°$

$$120^g + x = 180°$$
$$120^g \times \frac{9°}{10^g} + x = 108°$$
$$108° + x = 180°$$
$$x = 72°$$

Clave A

23. $\left(\frac{160n}{9}\right)^g + (14n)° = 90°$

$$\left(\frac{160n}{9}\right)^g \times \frac{9°}{10^g} + (14n)° = 90°$$
$$16n + 14n = 90$$
$$30n = 90$$
$$n = 3$$

Clave C

24. $x°y'z'' = 3°42'48'' + 5°29'34''$

Segundos: $48'' + 34'' = 82'' = 1' + \underbrace{22''}_{z}$

Minutos: $42' + 29' + 1' = 72' = 1° + \underbrace{12'}_{y}$

Grados: $3° + 5° + 1° = \underbrace{9°}_{x}$

Entonces: $x°y'z'' = 9°12'22''$

Piden:

$$E = \frac{z - y - 1}{x}$$
$$E = \frac{22 - 12 - 1}{9} = \frac{9}{9} = 1$$

Clave A

25. $x + y = 40^g \times \frac{9°}{10^g} = 36°$

$$x - y = \frac{\pi}{30}\text{ rad} \times \frac{180°}{\pi\text{ rad}} = 6°$$

Entonces:

$x + y = 36°$

$x - y = 6°$

Sumando:

$2x = 42° \Rightarrow x = 21°$

Clave C

Nivel 3 (página 9) Unidad 1

Comunicación matemática

26. I. Del triángulo isósceles:

$$a° = b^g \Rightarrow a° = \left(b.\frac{9}{10}\right)°$$
$$\Rightarrow a = \frac{9}{10}b$$
$$10a = 9b \qquad \dots (1)$$

De (1)

$$b = \frac{10a}{9} \Rightarrow b = a + \frac{a}{9}$$

$\therefore b > a$

I es falso

II. De (1)

II es verdadero

III. Del triángulo se cumple:

Si $y > x \Rightarrow \omega \text{ rad} > a°$

$$\omega \text{ rad} > a° . \frac{\pi \text{ rad}}{180°}$$
$$\omega \text{ rad} > \frac{a\pi}{180°} \text{ rad}$$
$$\omega > \frac{a\pi}{180°} \Rightarrow 180\,\omega > \pi a$$

$\therefore$ III es falso

Clave A

27. Completamos el recuadro:

Sexagesimal	Centesimal	Radial
36°	40^g	$\pi/5$ rad
171°	190^g	$19\,\pi/20$ rad
108°	120^g	$3\,\pi/5$ rad

Luego:

$$A = \left(\frac{3a - b + c}{\pi}\right)d$$

$a = 36$; $b = 108$; $c = 120$; $d = \frac{\pi}{5}$

Reemplazando en A:

$$A = \left(\frac{3(36) - 108 + 120}{\pi}\right)\left(\frac{\pi}{5}\right)$$
$$A = \left(\frac{108 - 108 + 120}{5}\right)$$
$$A = \frac{120}{5} \qquad \therefore A = 24$$

Clave E

Razonamiento y demostración

28. Del gráfico:

$$90° + x + 270^g = 360°$$
$$90° + x + 270^g \times \frac{9°}{10^g} = 360°$$
$$90° + x + 243° = 360°$$
$$x = 27°$$

Clave B

29. Del gráfico:

$$90° + \alpha - \beta + \theta + 70° = 360°$$
$$\alpha - \beta + \theta = 200°$$

Clave E

30. I. $48{,}5^g \; 47{,}8^m \; \underbrace{220^s}$

$$\underbrace{48{,}5^g + \underbrace{47{,}8^m + 2{,}2^m}_{0{,}5^g}}_{49^g}$$

Clave B

II. $43{,}2° \; 105{,}3' \; \underbrace{162''}$

$$\underbrace{43{,}2° + \underbrace{105{,}3' + 2{,}7'}_{1{,}8°}}_{45°}$$

Clave A

Resolución de problemas

31. Del gráfico:

$$\frac{\pi}{8}\text{ rad} + a^\circ b' = 180^\circ$$

$$\frac{\pi}{8}\text{ rad} \times \frac{180^\circ}{\pi\text{ rad}} + a^\circ b' = 180^\circ$$

$$\frac{45^\circ}{2} + a^\circ b' = 180^\circ$$

$$a^\circ b' = \frac{315^\circ}{2} = 157^\circ + 0{,}5^\circ$$

$$a^\circ b' = 157^\circ + 0{,}5^\circ \times \frac{60'}{1^\circ}$$

$$a^\circ b' = 157^\circ + 30'$$

Piden: $a + b = 157 + 30 = 187$

Clave C

32. De la condición:

$$\frac{SR}{C} = \frac{27\pi}{20}$$

$$\frac{9k\left(\frac{\pi}{20}k\right)}{10k} = \frac{27\pi}{20}$$

$$\frac{9\pi k^2}{200k} = \frac{27\pi}{20}$$

$$\frac{k}{10} = 3 \Rightarrow k = 30$$

Entonces:

$$N = \frac{9k + 10k}{57} = \frac{19k}{57} = \frac{k}{3}$$

$$N = \frac{30}{3} = 10$$

Clave E

33. $C + S^2 = 91$

$10k + (9k)^2 = 91$

$10k + 81k^2 = 91$

$81k^2 + 10k - 91 = 0$

$$\begin{array}{ccc} 81k & \times & 91 \\ 1k & & -1 \end{array}$$

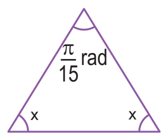
$\Rightarrow k = 1$

Se sabe:

$$R = \frac{\pi}{20}k = \frac{\pi}{20}(1) = \frac{\pi}{20}\text{ rad}$$

Clave E

34. Del gráfico:



$$2x + \frac{\pi}{15}\text{rad} = 180^\circ$$

$$2x + \frac{\pi}{15}\text{ rad} \times \frac{180^\circ}{\pi\text{ rad}} = 180^\circ$$

$$2x + 12^\circ = 180^\circ \Rightarrow x = 84^\circ$$

Clave B

35. $2S + C - \dfrac{20R}{\pi} = 27$

Sabemos:

$S = 9k$; $C = 10k$; $R = \dfrac{\pi}{20}k$

Luego:

$$2(9k) + 10k - \frac{20}{\pi}\left(\frac{\pi}{20}k\right) = 27$$

$18k + 10k - k = 27$

$27k = 27$

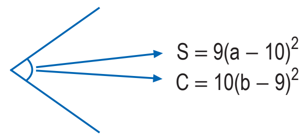
$k = 1$

$S = 9(1) = 9$

$\therefore$ La medida del ángulo es 9°.

Clave C

36.



$$\frac{S}{9} = \frac{C}{10} \Rightarrow \frac{9(a-10)^2}{9} = \frac{10(b-9)^2}{10}$$

$(a - 10)^2 = (b - 9)^2$

$(a - 10) = \pm(b - 9)$

- $a - 10 = b - 9 \Rightarrow a - b = 1$
- $a - 10 = -b + 9 \Rightarrow a + b = 19$

Piden:

$$E = \frac{a+b}{a-b} = \frac{19}{1} = 19$$

Clave D

SECTOR CIRCULAR

APLIQUEMOS LO APRENDIDO (página 11) Unidad 1

1. Datos:

$\theta = 80^g$

$D = 40\text{ cm} \Rightarrow R = 20\text{ cm}$

Piden:

$L = \theta \times R$

$L = \left(80^g \times \dfrac{\pi}{200^g}\right)(20\text{ cm})$

$L = 8\pi\text{ cm}$

Clave D

2. Datos:

$\dfrac{R}{L} = \dfrac{2}{3}$

Pero: $L = \theta \times R$

Reemplazando:

$\dfrac{R}{\theta \times R} = \dfrac{2}{3} \Rightarrow \theta = \dfrac{3}{2}\text{ rad}$

Clave D

3. Del gráfico:

$4\pi = 24°(10 + OC)$

$4\pi = 24 \times \dfrac{\pi}{180}(10 + OC)$

$4\pi = \dfrac{2\pi}{15} \times 10 + \dfrac{2\pi}{15} \times OC$

$4 = \dfrac{4}{3} + \dfrac{2}{15} \times OC$

$\Rightarrow \quad OC = 20$

Por lo tanto: $x = \dfrac{2\pi}{15} \times 20 = \dfrac{8\pi}{3}$

Clave C

4. Piden el área del sector circular:

$S = \dfrac{R^2\theta}{2}$

Datos: $\theta = 60° \times \dfrac{\pi}{180°} = \dfrac{\pi}{3}$

$R = 4\text{ cm}$

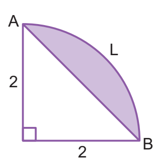
Reemplazando:

$S = \dfrac{16 \times \dfrac{\pi}{3}}{2}$

$S = \dfrac{8\pi}{3}\text{ cm}^2$

Clave B

5.



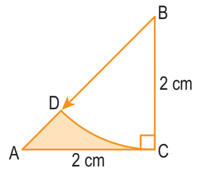
Del gráfico: $L = \dfrac{\pi}{2} \times 2 = \pi$

$AB = 2\sqrt{2}$

Piden: $m\overline{AB} + m\overset{\frown}{AB} = 2\sqrt{2} + \pi$

Clave B

6. Del gráfico:



Piden:

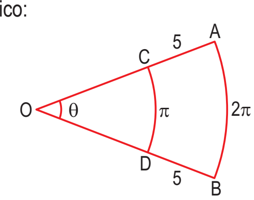
$S_{somb.} = S_{\triangle ACB} - S_{\sphericalangle DBC}$

$S_{somb.} = \dfrac{2 \times 2}{2} - \dfrac{\pi}{4} \times 2$

$S_{somb.} = \left(2 - \dfrac{\pi}{2}\right)\text{cm}^2$

Clave E

7. Del gráfico:



Piden: θ

Pero $L = \theta \times R$

$L_{\overset{\frown}{CD}} = \pi = \theta \times OC$ ...(1)

$L_{\overset{\frown}{AB}} = 2\pi = \theta \times (5 + OC)$ ...(2)

(1) : (2):

$\dfrac{\pi}{2\pi} = \dfrac{\theta \times OC}{\theta(5 + OC)}$

$\dfrac{1}{2} = \dfrac{OC}{5 + OC}$

$5 + OC = 2OC \Rightarrow OC = 5$

En (1):

$\pi = \theta \times 5 \Rightarrow \theta = \dfrac{\pi}{5}\text{ rad}$

Clave A

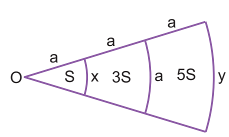
8. Del gráfico:

$\dfrac{\pi}{6} = 30° \times \dfrac{\pi}{180°} \times r$

$\dfrac{\pi}{6} = \dfrac{\pi}{6} \times r \Rightarrow r = 1\text{ m}$

Clave A

9.



$S = \dfrac{ax}{2}$ ...(1)

$4S = \dfrac{2a \times a}{2}$ ...(2)

$9S = \dfrac{3ay}{2}$ ...(3)

(1) : (2):

$\dfrac{S}{4S} = \dfrac{\dfrac{ax}{2}}{\dfrac{2a^2}{2}}$

$\Rightarrow \dfrac{1}{4} = \dfrac{x}{2a} \Rightarrow x = \dfrac{a}{2}$

(2) : (3):

$\dfrac{4S}{9S} = \dfrac{\dfrac{2a^2}{2}}{\dfrac{3ay}{2}}$

$\dfrac{4}{9} = \dfrac{2a}{3y} \Rightarrow y = \dfrac{3a}{2}$

Piden:

$x + y = \dfrac{a}{2} + \dfrac{3a}{2} = 2a$

Clave D

10. De los datos:

$L = 4\pi\text{ cm}$

$R = 9\text{ cm}$

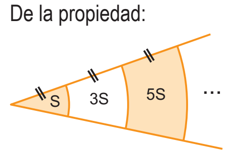
$L = \theta \times R$

$4\pi = \theta \times 9 \Rightarrow \theta = \dfrac{4\pi}{9} \times \dfrac{180°}{\pi}$

$\theta = 80°$

Clave E

11. De la propiedad:



Entonces

$S_1 = S;\ S_2 = 3S$

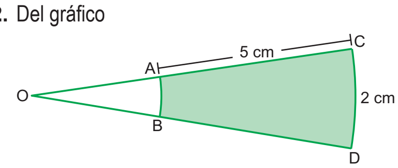
En el problema:

$\dfrac{2S_2 - 3S_1}{S_2 + S_1} = \dfrac{2(3S) - 3(S)}{3S + S} = \dfrac{3S}{4S} = \dfrac{3}{4}$

$\therefore \dfrac{2S_2 - 3S_1}{S_2 + S_1} = \dfrac{3}{4} = 0{,}75$

Clave C

12. Del gráfico



De la propiedad área del trapecio:

$S = \dfrac{(a + b)}{2}h$ ...(1)

Datos:

$a = 2\text{cm},\ h = 5\text{cm};\ S = 8\text{cm}^2;\ b = x$

Reemplazando en (1)

$8 = \dfrac{(2 + x)}{2}(5)$

$\dfrac{16}{5} = (2 + x)$

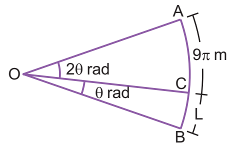
$x = \dfrac{16}{5} - 2$

$x = \dfrac{6}{5} = 1{,}2$

$\therefore x = 1{,}2\text{ cm}$

Clave E

13.



Sabemos:

$L = \theta \,.\, R \Rightarrow R = \dfrac{L}{\theta}$

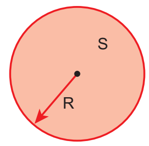
Los sectores circulares AOC y COD tienen igual radio entonces:

$R = \dfrac{9\pi}{2\theta} = \dfrac{L}{\theta} \Rightarrow \dfrac{9\pi}{2} = L$

$\therefore\ L = 4{,}5\pi$ m

Clave C

14. Del enunciado:

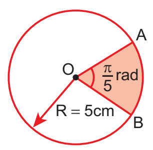


Dato:

$S = \pi R^2 = 25\pi\ cm^2 \Rightarrow R^2 = 25$

$R = 5$ cm

Luego; sea el sector circular AOB;



$L_{\widehat{AB}} = \theta \,.\, R$

$L_{\widehat{AB}} = \dfrac{\pi}{5}.5$

$L_{\widehat{AB}} = \pi \qquad \therefore\ L_{\widehat{AB}} = \pi$ cm

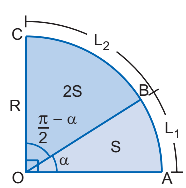
Clave D

PRACTIQUEMOS

Nivel 1 (página 13) Unidad 1

Comunicación matemática

1. Del gráfico:



$S = \dfrac{1}{2}\ \alpha R^2; \qquad 2S = \dfrac{1}{2}\left(\dfrac{\pi}{2} - \alpha\right)R^2$

De las proposiciones

I. $2\left(\dfrac{1}{2}\alpha R^2\right) = \dfrac{1}{2}\left(\dfrac{\pi}{2} - \alpha\right)R^2$

$2\alpha = \dfrac{\pi}{2} - \alpha$

$3\alpha = \dfrac{\pi}{2} \Rightarrow \alpha = \dfrac{\pi}{6}$

$\therefore\ \alpha$ es igual a $\dfrac{\pi}{6}$ rad.

I. falsa

II. De la expresión del área:

$S = \dfrac{1}{2}\ \alpha R^2;\ \ \alpha = \dfrac{\pi}{6}$ (de lo anterior)

$S = \dfrac{1}{2}\dfrac{\pi}{6}\ R^2 = \dfrac{\pi R^2}{12}$

Para $R = 2$ m

$S = \dfrac{\pi}{12}(2)^2 = \dfrac{\pi}{3}$

$\therefore$ S es igual a $\dfrac{\pi}{3}\ m^2$

II. Verdadera

III. Usando la expresión del área $\dfrac{RL}{2}$:

$S = \dfrac{L_1}{2} \quad ; \quad 2S = \dfrac{RL_2}{2}$

De las dos expresiones:

$\Rightarrow 2\left(\dfrac{RL_1}{2}\right) = \dfrac{RL_2}{2} \rightarrow 2L_1 = L_2$

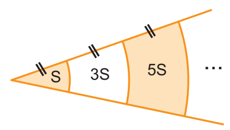
$L_1 = \dfrac{L_2}{2}$

$\therefore\ L_1$ es la mitad de L_2

III. verdadera

Clave C

2. De la figura podemos observar por propiedad:



Luego:

$S_1 = S\ ;\ S_2 = 3S;\ S_3 = 5S$

Entonces se observa:

$2S_1 + S_2 = S_3$

$\therefore$ La proposición E:

El doble de S_1 más S_2 es igual a S_3.

Verdadera

Clave E

Razonamiento y demostración

3. Del gráfico:

$\theta = 45° = \dfrac{\pi}{4}$ rad

$R = 16$ m

Entonces:

$L = \theta \times R$

$L = \dfrac{\pi}{4} \times 16$ m

$L = 4\pi$ m

Clave D

4. Del gráfico:

$\theta = 80° = \dfrac{4\pi}{9}$ rad

$L = 24\pi$ m

Entonces:

$L = \theta \times R$

$24\pi\ m = \dfrac{4\pi}{9} \times R$

$R = 54$ m

Clave E

5. $\theta = 40^g \times \dfrac{\pi\ rad}{200^g} = \dfrac{\pi}{5}$ rad

$R = 10$ m

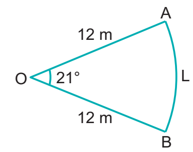
Piden L:

$L = \theta \times R = \dfrac{\pi}{5} \times 10$

$L = 2\pi$ m

Clave B

6.



21° a radianes:

$\dfrac{S}{9} = \dfrac{20R}{\pi}$

$\dfrac{21}{9} = \dfrac{20R}{\pi}$

$R = \dfrac{7\pi}{60} \Rightarrow \theta = \dfrac{7\pi}{60}$ rad

Como: $L = \theta\ .\ r$

$L = \dfrac{7\pi}{60}.12 = \dfrac{7}{5}\pi = \dfrac{7}{5} \times \dfrac{22}{7} = \dfrac{22}{5}$ m

$\therefore\ L = \dfrac{22}{5}$ m

Clave D

7. Área del sector circular:

$S = \dfrac{\theta\,.R^2}{2}$

$\theta = 150^g\left(\dfrac{\pi\ rad}{200^g}\right) = \dfrac{3\pi}{4}$ rad

$R = 8$ cm

$S = \dfrac{\left(\dfrac{3\pi}{4}\right)(8)^2}{2} = 24\pi$

Clave A

8. Área del sector circular:

$S = \dfrac{\theta\ .\ R^2}{2}$

$\left.\begin{array}{l} \theta = 45° = \dfrac{\pi}{4}\ \text{rad} \\ R = 6\sqrt{2}\ \text{m} \end{array}\right\}$

$S = \dfrac{\left(\dfrac{\pi}{4}\right)(6\sqrt{2})^2}{2} = 9\pi$

Clave A

9. Área del sector circular: $S = \dfrac{\theta\ .\ R^2}{2}$

$R = 16$ m

$\theta = 50^g\left(\dfrac{\pi\ rad}{200^g}\right) = \dfrac{\pi}{4}$ rad

$$S = \frac{\left(\frac{\pi}{4}\right)(16)^2}{2}$$

$$S = \frac{64\pi}{2} = 32\pi\ m^2$$

Clave B

10. Área del sector circular: $S = \frac{\theta . R^2}{2}$

$R = 18$ m

$$\theta = 70°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{7\pi}{18} \text{ rad}$$

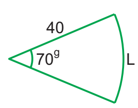
$$S = \frac{\left(\frac{7\pi}{18}\right)(18)^2}{2}$$

$$S = \frac{126\pi}{2} = 63\pi\ m^2$$

Clave B

Resolución de problemas

11.



70^g a radianes:

$$\frac{C}{10} = \frac{20R}{\pi}$$

$$\frac{70}{10} = \frac{20R}{\pi}$$

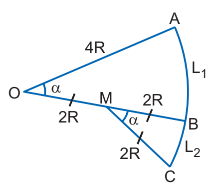
$$R = \frac{7\pi}{20} \Rightarrow \theta = \frac{7\pi}{20} \text{ rad}$$

$$\Rightarrow L = \frac{7\pi}{20} \times 40$$

$\therefore L = 14\pi$ cm

Clave B

12.



Por dato: $\alpha = \frac{\pi}{6}$ rad

Entonces:

$$L_1 = \alpha . 4R = \left(\frac{\pi}{6}\right) . 4R = \frac{2\pi R}{3}$$

$$\Rightarrow L_1 = \frac{2\pi R}{3}$$

$$L_2 = \alpha . 2R = \left(\frac{\pi}{6}\right) . 2R = \frac{\pi R}{3}$$

$$\Rightarrow L_2 = \frac{\pi R}{3}$$

Piden:

$$L_1 + L_2 = \frac{2\pi R}{3} + \frac{\pi R}{3} = \pi R$$

$$\therefore L_1 + L_2 = \pi R$$

Clave A

13. Por dato:

$$\theta = 62^g . \left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{31\pi}{100} \text{ rad}$$

$R = 1$ m

Piden: la longitud del arco (L).

$$L = \theta . R = \left(\frac{31\pi}{100}\right)(1)$$

$$\Rightarrow L = \frac{31\pi}{100} \text{ m} = 31\pi \text{ cm}$$

$\therefore L = 31\pi$ cm

Clave D

14. Por dato:

$$\theta = 30° = \frac{\pi}{6} \text{ rad}$$

$R = 12$ cm

Piden: la longitud del arco (L).

$$L = \theta . R = \left(\frac{\pi}{6}\right)(12) = 2\pi$$

$\therefore L = 2\pi$ cm

Clave B

15. Por dato:

$L = 3\pi$ cm

$$\theta = 60^g = \frac{3\pi}{10} \text{ rad}$$

Piden: la medida del radio (R).

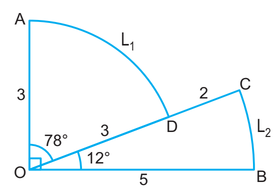
Se cumple: $L = \theta . R$

$$\Rightarrow 3\pi = \left(\frac{3\pi}{10}\right)R \Rightarrow R = 10$$

$\therefore R = 10$ cm

Clave B

16.



Del gráfico:

$$L_1 = \left(78° . \frac{\pi \text{ rad}}{180°}\right)(3) = \frac{13\pi}{10}$$

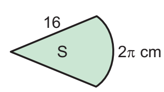
$$L_2 = \left(12° . \frac{\pi \text{ rad}}{180°}\right)(5) = \frac{\pi}{3}$$

Piden:

$$J = \frac{L_1}{L_2} = \frac{\frac{13\pi}{10}}{\frac{\pi}{3}} = \frac{39}{10} \qquad \therefore J = 3{,}9$$

Clave B

17.



$$S = \frac{L . r}{2}$$

$$S = \frac{2\pi . 16}{2}$$

$S = 16\pi\ cm^2$

Clave A

18. Por dato:

$$\theta = 30° = \frac{\pi}{6} \text{ rad}$$

$R = 2\sqrt{3}$ cm

Piden: el área del sector circular (A).

$$A = \frac{\theta . R^2}{2} = \frac{\left(\frac{\pi}{6}\right)(2\sqrt{3})^2}{2} = \pi$$

$\therefore A = \pi\ cm^2$

Clave A

19. Por dato:

$L = 2\pi$ cm

$R = 8$ cm

Piden: la medida del ángulo central (θ).

$L = \theta . R$

$2\pi = \theta . 8$

$$\Rightarrow \theta = \frac{\pi}{4} \text{ rad} . \left(\frac{200^g}{\pi \text{ rad}}\right)$$

$\therefore \theta = 50^g$

Clave B

Nivel 2 (página 14) Unidad 1

Comunicación matemática

20. I. De la proposición:

$$\frac{\pi}{3}R = L$$

Por definición de longitud de arco:

L es igual al producto del radio por el número de radianes del ángulo AOB, por lo tanto:

$$m < AOB = \frac{\pi}{3} \text{ rad}$$

Transformando al sistema sexagesimal:

$$m < AOB = \frac{\pi}{3} \text{ rad}.1 = \frac{\pi}{3} \text{ rad}.\frac{180°}{\pi \text{ rad}}$$

$m < AOB = 60°$

I. Verdadera

II. De la proposición:

θ es igual al número de grados sexagesimales del ángulo AOB, luego:

$$\theta° = \theta° . \frac{\pi \text{ rad}}{180°} = \frac{\pi\theta}{180} \text{ rad}$$

$\frac{\pi\theta}{180}$ es el número de radianes del ángulo AOB, por lo tanto:

$$L = \frac{\pi\theta}{180} \text{ rad}$$

II. Falsa

III. De la proposición:

Sea el perímetro de AOB 2p:

$2p = 2R + L$

L: longitud de arco AB

Por condición:

$2p = 5R = 2R + L$

Luego:

$L = 3R$

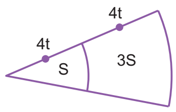
$$R = \frac{L}{3}$$

El radio (R) es igual a la tercera parte de la longitud de arco (L)

III. Falsa

Clave E

21. Por propiedad:



Entonces:

$S_2 = 3S_1$ ... (1)

Por dato: $S_2 = S_3$

En (1)

$S_3 = 3S_1$ ... (2)

$\therefore$ S_3 es el triple de S_1

Además:

$S_1 = \frac{1}{2}\theta R^2;\ S_3 = \frac{1}{2}\alpha R^2$

En (2)

$\frac{1}{2}\alpha R^2 = 3\frac{1}{2}\theta R^2$

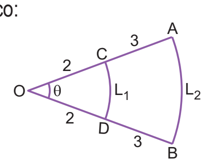
$\alpha = 3\theta$

$\therefore$ α es igual al triple de θ

Clave C

Razonamiento y demostración

22. Del gráfico:



$L_1 = \theta \,.\, 2$

$L_1 = 2\theta$

$L_2 = \theta \,.\, 5$

$L_2 = 5\theta$

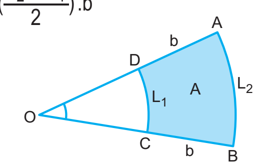
$$J = \frac{3L_1 + L_2}{L_2} = \frac{3(2\theta)+5\theta}{5\theta} = \frac{11\theta}{5\theta}$$

$\therefore\ J = 2{,}2$

Clave D

23. Área del trapecio circular:

$$A = \left(\frac{L_2+L_1}{2}\right).b$$

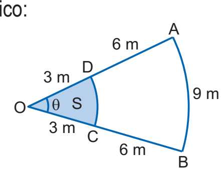


Para:

$$\left.\begin{array}{l} L_2 = 2\pi \\ L_1 = \pi \\ \beta = 5 \end{array}\right\} A = \left(\frac{2\pi+\pi}{2}\right).5 = \frac{15\pi}{2}\ m^2$$

Clave E

24. Del gráfico:



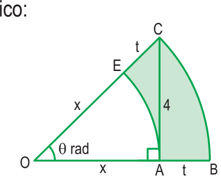
$L_{\overset{\frown}{AB}} = 9 = 9 \times \theta \Rightarrow \theta = 1$ rad

Piden: $S_{\triangleleft DOC} = \frac{R^2\theta}{2}$

$$S_{\triangleleft DOC} = \frac{(3)^2 \times 1}{2} = 4{,}5\ m^2$$

Clave B

25. Del gráfico:



En el ⊾OAC, por el teorema de Pitágoras:

$x^2 + 16 = (x+t)^2$

$x^2 + 16 = x^2 + 2xt + t^2$

$2xt + t^2 = 16$

$t(t + 2x) = 16$

Los valores que cumplen son:

$2(2 + 2(3)) = 16$

$\therefore\ t = 2 \ \wedge\ x = 3$

$A_{somb.} = S_{\triangleleft BOC} - S_{\triangleleft EOA}$

$$A_{somb.} = \frac{(x+t)^2\theta}{2} - \frac{x^2\theta}{2}$$

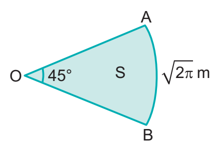
$$A_{somb.} = \frac{(3+2)^2\theta}{2} - \frac{3^2\theta}{2}$$

$$A_{somb.} = \frac{25\theta}{2} - \frac{9\theta}{2}$$

$A_{somb.} = 8\theta$

Clave D

26.



45° a radianes:

$$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{45}{9} = \frac{20R}{\pi} = \theta$$

$R = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$ rad

$$S = \frac{L^2}{2\theta} = \frac{\sqrt{2\pi}^2}{2\left(\frac{\pi}{4}\right)} = \frac{2\pi}{\frac{\pi}{2}} = 4\ m^2$$

Clave C

Resolución de problemas

27. Dato:

$D = 48$ m $\Rightarrow$ $r = 24$ m

$\theta = 60°$

60°a radianes:

$$\frac{S}{9} = \frac{20R}{\pi}$$

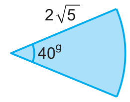
$$\frac{60}{9} = \frac{20R}{\pi} \Rightarrow R = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}\ \text{rad}$$

$L = \theta \,.\, r$

$L = \theta \,.\, r = \frac{\pi}{3} \,.\, 24 = 8\pi$ m

Clave C

28.



$$\frac{S}{9} = \frac{C}{10} = \frac{20R}{\pi}$$

40^g a radianes:

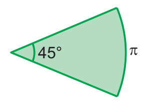
$$\frac{40}{10} = \frac{20R}{\pi}$$

$R = \frac{\pi}{5} \Rightarrow \theta = \frac{\pi}{5}$ rad

$$S = \frac{\theta\,.\,r^2}{2} = \frac{\pi}{5}\frac{(2\sqrt{5})^2}{2} = \frac{20\pi}{10} = 2\pi\ cm^2$$

Clave B

29.



45° a radianes:

$$\frac{S}{9} = \frac{20R}{\pi}$$

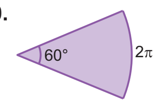
$$\frac{45}{9} = \frac{20R}{\pi}$$

$\Rightarrow R = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$ rad

$$S = \frac{L^2}{2\theta} \Rightarrow S = \frac{\pi^2}{2\left(\frac{\pi}{4}\right)} = 2\pi\ cm^2$$

Clave B

30.



60° a radianes:

$$\frac{S}{9} = \frac{20R}{\pi}$$

$$\frac{60}{9} = \frac{20R}{\pi} = \theta$$

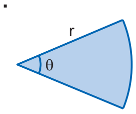
$R = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$ rad

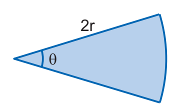
$$S = \frac{L^2}{2\theta}$$

$$S = \frac{(2\pi)^2}{\frac{2.\pi}{3}} = \frac{4\pi^2\,.\,3}{2\pi} = 6\pi\ cm^2$$

Clave E

31.





$$S = \frac{\theta\,.\,r^2}{2}$$

$$S_2 = \frac{\theta.(2r)^2}{2}$$

$$S_2 = 4\left(\frac{\theta r^2}{2}\right)$$

$S_2 = 4S$

$\therefore$ Aumenta en 3S.

Clave C

Nivel 3 (página 15) Unidad 1

Comunicación matemática

32. I. De la figura:

$$m\angle AOB = 20^g = 20^g\,.\,\frac{\pi\ \text{rad}}{200^g} = \frac{\pi}{10}$$

$m\angle AOB = \frac{\pi}{10}$ rad

Además:
$S_{\triangleleft AOB} = \frac{(L_{\widehat{AB}})^2}{2\theta}$; θ: número de radianes del ángulo AOB.

Entonces; del gráfico:

$$S_{\triangleleft AOB} = \frac{(3\pi)^2}{2\left(\frac{\pi}{10}\right)} = 45\pi$$

$\therefore\ S_{\triangleleft AOB} = 45\pi\ cm^2$

Nota: unidades de $L_{\widehat{AB}}$ son centímetros (cm), por lo tanto: unidades de $S_{\triangleleft AOB}$ cm^2

II. Se observa:
$m\angle CO'D = 35° = 35° \cdot \frac{\pi\ rad}{180°} = \frac{7\pi}{36}$ rad

Luego:
$S_{\triangleleft CO'D} = \frac{\alpha . R^2}{2}$;

R: radio.
α: n.° de radianes del ángulo CO'D.

Reemplazando:
$$S_{\triangleleft CO'D} = \frac{1}{2} \cdot \frac{7\pi}{36}\ (6)^2 = \frac{7\pi}{2}$$

$\therefore S_{CO'D} = \frac{7\pi}{2}\ cm^2$

III. De los datos:
$$S_{\triangleleft EOF} = \frac{L_{\widehat{EF}} . R}{2}$$

Reemplazando
$$S_{\triangleleft EOF} = \frac{7 . \pi}{2}$$

$\therefore\ S_{\triangleleft EOF} = \frac{7\pi}{2}\ m^2$

Clave E

33. De la figura:

$S_1 = \frac{1}{2}\ \theta R^2$; $S_2 = \frac{1}{2}\ \alpha(3R)^2 - \frac{1}{2}\alpha(2R)^2$

$S_2 = \frac{1}{2}\ \alpha\ .\ 5R^2 = \frac{5}{2}\ \alpha R^2$

De la condición:

$S_1 = S_2$

$\frac{1}{2}\theta R^2 = \frac{5}{2}\ \alpha R^2$

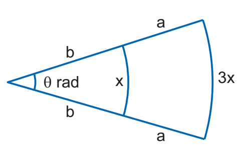
$\theta = 5\alpha$

Por proporciones:
α es a θ como 1 es a 5

Clave E

Razonamiento y demostración

34.

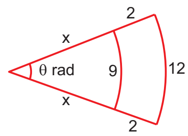


$L = \theta\ .\ R$
$x = \theta\ .\ b$
$3x = \theta(a + b)$

$\frac{1}{3} = \frac{b}{(a+b)} \Rightarrow a + b = 3b$
$a = 2b$
$\therefore \frac{a}{b} = 2$

Clave A

35.



Por la propiedad:

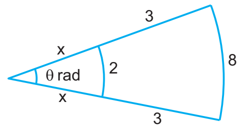
$\theta = \frac{12-9}{2} = \frac{3}{2}$

$L = \theta R$

$9 = \frac{3}{2}\ .\ x \Rightarrow x = 6$

Clave B

36.



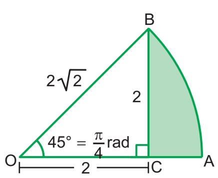
$L = \theta\ .\ R$

$$\left.\begin{aligned} 8 &= \theta\ .\ (x+3) \\ 2 &= \theta\ .\ x \end{aligned}\right\}\ \text{dividir}$$

$\frac{8}{2} = \frac{x+3}{x} \Rightarrow x = 1$

Clave A

37. Área del sector: $S = \frac{\left(\frac{\pi}{4}\right)(2\sqrt{2})^2}{2} = \pi$

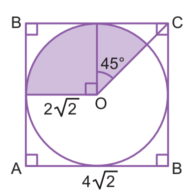


Área del triángulo: $A = \frac{2.2}{2} = 2$

$\Rightarrow A_{somb.} = A_{\triangleleft BOA} - A_{\triangle OCB}$
$A_{Somb.} = S - A$
$A_{Somb.} = \pi - 2$

Clave B

38. Del gráfico:



$R = 2\sqrt{2}$ m

$\theta = 90° + 45° = 135° \times \frac{\pi\ rad}{180°} = \frac{3\pi}{4}$ rad

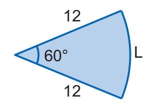
$$S_{somb.} = \frac{(2\sqrt{2})^2\left(\frac{3\pi}{4}\right)}{2}$$

$S_{somb.} = 3\pi\ m^2$

Clave C

Resolución de problemas

39.



60° a radianes:
$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{60}{9} = \frac{20R}{\pi}$

$R = \pi/3 \Rightarrow \theta = \frac{\pi}{3}$ rad
$L = \frac{\pi}{3} \times 12 = 4\pi$

Por lo tanto:
Perímetro = $12 + 12 + 4\pi = 4\pi + 24$
$\therefore$ Perímetro = $4(6 + \pi)$

Clave D

40. $\theta = 36° \Rightarrow \theta = \frac{\pi}{5}$ rad

1.^{er} caso: ángulo θ y radio R
2.° caso: ángulo α y radio $\frac{3}{4}$R
Por dato el área no varía:
$S_{1^{er}.\ caso} = S_{2^{o}.\ caso}$

$\theta\ .\ R^2 = \alpha\left(\frac{3}{4}R\right)^2 \Rightarrow \theta\ .\ R^2 = \alpha\ .\ \frac{9}{16}R^2$

$\Rightarrow \alpha = \frac{16}{9}\theta$

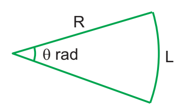
Como $\theta = \frac{\pi}{5}$ rad, entonces:

$\alpha = \frac{16}{9}\left(\frac{\pi}{5}\right) = \frac{16\pi}{45}$ rad $= 64°$

$\therefore$ Lo que hay que aumentar es:
$64° - 36° = 28°$

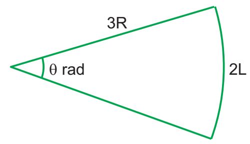
Clave A

41. Área del sector circular es $12\pi\ cm^2$:



$\Rightarrow \frac{L.R}{2} = 12\pi$
$\Rightarrow L\ .\ R = 24\pi$

El arco se duplica y el radio se triplica:



Entonces:

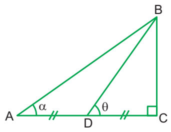
$S = \frac{(2L)(3R)}{2}$

$S = 3\underbrace{L.R}_{24\pi} = 72\pi$

Clave E

APLIQUEMOS LO APRENDIDO (página 17) Unidad 1

1. Sea el gráfico:



$\cot\theta = \dfrac{\frac{AC}{2}}{BC}$ $\quad \tan\alpha = \dfrac{BC}{AC}$

Reemplazando en E:
$E = \cot\theta\tan\alpha$

$E = \dfrac{\frac{AC}{2}}{BC} \times \dfrac{BC}{AC} \Rightarrow E = \dfrac{1}{2}$

Clave A

2. Del gráfico, por el teorema de Pitágoras:

$AB^2 = AC^2 + BC^2$

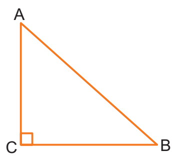
$16 = 9 + BC^2 \;\Rightarrow\; BC = \sqrt{7}$

Piden $\tan\alpha$:

$\tan\alpha = \dfrac{\sqrt{7}}{3}$

Clave B

3. Sea el triángulo:



Condición:
$\cos B - \cos A = 2\operatorname{sen} B$

$\dfrac{CB}{AB} - \dfrac{AC}{AB} = 2\dfrac{AC}{AB}$

$CB - AC = 2AC$

$CB = 3AC \Rightarrow \dfrac{1}{3} = \dfrac{AC}{CB} = \cot A$

Entonces: $\cot A = \dfrac{1}{3}$

Clave D

4. Del gráfico:

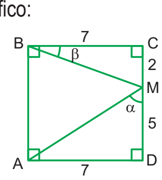
$\tan\theta = \dfrac{CB}{5\text{ m}}$ $\quad \tan\alpha = \dfrac{2\text{ m}}{CB}$

Piden:
$E = \tan\theta\tan\alpha$

$E = \dfrac{CB}{5\text{ m}} \times \dfrac{2\text{ m}}{CB} \;\Rightarrow E = \dfrac{2}{5}$

Clave C

5. En el gráfico:



Dato: $\cot\alpha = \dfrac{5}{7}$

$\Rightarrow CM = 2$

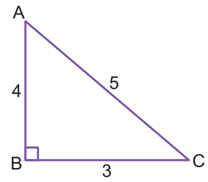
Piden $\cot\beta$:

$\cot\beta = \dfrac{BC}{CM} = \dfrac{7}{2}$

$\Rightarrow \cot\beta = 3{,}5$

Clave E

6. Sea el gráfico:



$\operatorname{sen} A = 0{,}6 = \dfrac{6}{10} = \dfrac{3}{5} = \dfrac{BC}{AC}$

Por el teorema de Pitágoras: $AB = 4$

Calculando M:

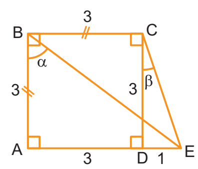
$M = \sec C + \cot A$

$M = \dfrac{5}{3} + \dfrac{4}{3} = 3$

$\therefore M = 3$

Clave B

7. Según el gráfico:



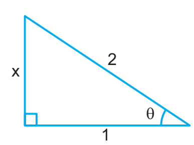
$\tan\beta = \dfrac{DE}{CD} = \dfrac{1}{3}$

Piden: $\cot\alpha = \dfrac{3}{4}$

Clave C

8. $\cos\theta = 0{,}5 = \dfrac{5}{10} = \dfrac{1}{2}$

Entonces:



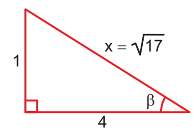
Por el teorema de Pitágoras:
$x^2 = 2^2 - 1^2$
$x = \sqrt{3}$

Luego:
$\theta = \sqrt{3} + 2\dfrac{\sqrt{3}}{2} \;\Rightarrow\; \theta = 2\sqrt{3}$

Clave B

9. $\tan\beta = 0{,}25 = \dfrac{25}{100} = \dfrac{1}{4}$

Entonces:



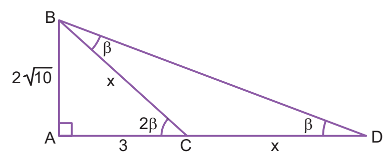
$x^2 = 4^2 + 1^2 \Rightarrow\; x = \sqrt{17}$

Luego:
$E = \dfrac{\operatorname{sen}\beta + \cos\beta}{\csc\beta - \sec\beta}$

$E = \dfrac{\frac{1}{\sqrt{17}} + \frac{4}{\sqrt{17}}}{\frac{\sqrt{17}}{1} - \frac{\sqrt{7}}{4}} = \dfrac{\frac{5}{\sqrt{17}}}{\frac{5\sqrt{17}}{4}} = \dfrac{4}{17}$

Clave C

10. En el gráfico:



Trazamos un segmento de tal modo que $BC = CD$, formando el triángulo isósceles BCD.

Entonces: $\cot\beta = \dfrac{3 + x}{2\sqrt{10}}$

Por el teorema de Pitágoras:
$BC = x = \sqrt{\left(2\sqrt{10}\right)^2 + (3)^2} = \sqrt{49} = 7$

Reemplazando:
$\cot\beta = \dfrac{10}{2\sqrt{10}} = \dfrac{\sqrt{10}}{2}$

Clave B

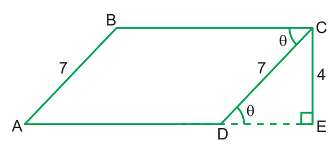
11. En el ⊿ABC, por el teorema de Pitágoras:
$AC^2 = 4^2 + 8^2$
$AC^2 = 16 + 64$
$AC^2 = 80$
$AC = 4\sqrt{5}$

Nos piden $\operatorname{sen}\alpha$:
En el ⊿ADC

$\operatorname{sen}\alpha = \dfrac{AD}{AC} = \dfrac{3\sqrt{5}}{4\sqrt{5}} \therefore \operatorname{sen}\alpha = \dfrac{3}{4}$

Clave D

12. Por los datos:



$\overline{CE}$: distancia entre $\overline{BC}$ y $\overline{AD}$

$AB = DC$

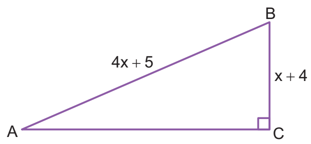
Luego:

$\text{sen}\theta = \dfrac{EC}{AC} = \dfrac{4}{7}$

$\therefore \text{sen}\beta = \dfrac{4}{7}$

Clave C

13. Por dato:



$\text{senA} = \dfrac{12}{37}$

En el ◺ADC

$\text{senA} = \dfrac{x+4}{4x+5} = \dfrac{12}{37}$

Luego:

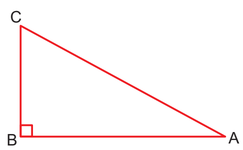
$37(x + 4) = (4x + 5)12$

$37x + 148 = 48x + 60$

$88 = 11x \qquad \therefore\ x = 8$

Clave B

14. Sea el ◺ABC:



Por dato: $\text{senA} = \dfrac{8}{17} = \dfrac{BC}{AC}$

$\Rightarrow\ BC = 8k,\ AC = 17k$

Por el teorema de Pitágoras: $\overline{AB}^2 + \overline{BC}^2 = \overline{AC}^2$

$AB^2 + (8k)^2 = (17k)^2$

$AB^2 = (17k)^2 - (8k)^2$

$AB^2 = 225k^2$

$AD = 15k$

Nos piden cscC

$\text{cscC} = \dfrac{AC}{AB} = \dfrac{17k}{15k} \qquad \therefore\ \text{cscC} = \dfrac{17}{15}$

Clave E

PRACTIQUEMOS
Nivel 1 (página 19) Unidad 1

Comunicación matemática

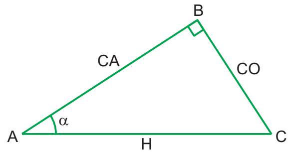
1. I. En todo triángulo rectángulo, se cumple el teorema de Pitágoras. ... (V)

II. En el triángulo rectángulo se definen 3 ángulos; 2 agudos y un ángulo recto (90°). ... (F)

III. Al lado AC se le opone el ángulo recto; pro teorema de correspondencia en el triángulo: a mayor ángulo, mayor lado. Por lo tanto $\overline{AC}$ es el mayor de los lados (hipotenusa) en el triángulo rectángulo.

Clave C

2. Sea un triángulo rectángulo con α ángulo agudo.



a) Cateto opuesto sobre hipotenusa es la definición de seno.

$\text{sen}\alpha = \dfrac{CO}{H}$

b) Cociente de la hipotenusa entre cateto adyacente; definición de secante para un ángulo.

$\sec\alpha = \dfrac{H}{CA}$

c) Por teorema de correspondencia, el mayor lado en un triángulo rectángulo es la hipotenusa ya que se le opone al ángulo recto.

Clave C

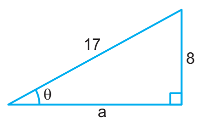
Razonamiento y demostración

3. Por el teorema de Pitágoras:

$17^2 = a^2 + 8^2$

$289 = a^2 + 64$

$a = 15$



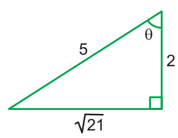
Piden: $\cos\theta = \dfrac{CA}{H} = \dfrac{15}{17}$

Clave A

4.

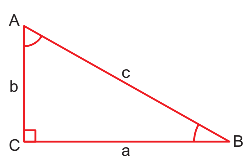
$\tan\theta = \dfrac{CO}{CA}$

$\tan\theta = \dfrac{\sqrt{21}}{2}$



Clave D

5.



$\text{senA} = \dfrac{CO}{H} = \dfrac{a}{c}$

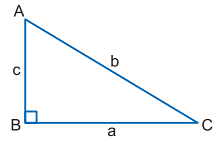
$\text{senB} = \dfrac{H}{CA} = \dfrac{c}{a}$

Piden: M = senA . secB

$M = \left(\dfrac{a}{c}\right)\left(\dfrac{c}{a}\right) = 1$

Clave A

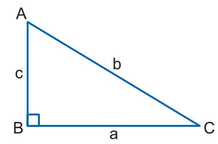
6. Sea el triángulo rectángulo:



$L = \text{senC}.\sec A = \dfrac{c}{b} \times \dfrac{b}{c} = 1$

Clave C

7.



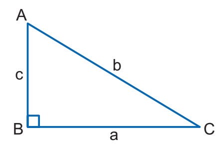
L = secAsecCsenCsenA

$L = \dfrac{b}{c} \times \dfrac{b}{a} \times \dfrac{c}{b} \times \dfrac{a}{b}$

L = 1

Clave C

8.



$L = (\sec^2A - \cot^2C)(\csc^2C - \tan^2A)$

$L = \left[\left(\dfrac{b}{c}\right)^2 - \left(\dfrac{a}{c}\right)^2\right]\left[\left(\dfrac{b}{c}\right)^2 - \left(\dfrac{a}{c}\right)^2\right]$

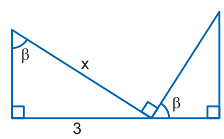
$L = \left(\dfrac{b^2}{c^2} - \dfrac{a^2}{c^2}\right)\left(\dfrac{b^2}{c^2} - \dfrac{a^2}{c^2}\right)$

$L = \dfrac{b^2 - a^2}{c^2} \times \dfrac{b^2 - a^2}{c^2}$

$L = \dfrac{c^2}{c^2} \times \dfrac{c^2}{c^2} \qquad \therefore L = 1$

Clave A

9.

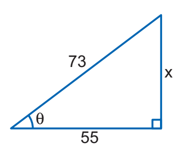


$\text{sen}\beta = \dfrac{\sqrt{3}}{2} = \dfrac{3}{x} \Rightarrow x = \dfrac{6}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right) = \dfrac{6\sqrt{3}}{3}$

$\therefore x = 2\sqrt{3}$

Clave E

10.



Por el teorema de Pitágoras:

$x^2 + 55^2 = 73^2$

$x^2 + 3025 = 5329$

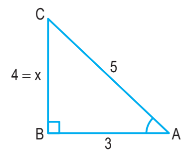
$x^2 = 2304 \Rightarrow x = 48$

$\Rightarrow P = \sec\theta - \tan\theta = \frac{73}{55} - \frac{48}{55}$

$\therefore P = \frac{25}{55} = \frac{5}{11}$

Clave A

11.



$x^2 + 3^2 = 5^2$

$x^2 = 16$

$\Rightarrow x = 4$

Luego:

$$E = \frac{12\left(\frac{4}{3} + \frac{3}{4}\right)}{5\left(\frac{5}{3}\right)} = \frac{12\left(\frac{25}{12}\right)}{\frac{25}{3}} = 3$$

Clave A

Resolución de problemas

12. Por el teorema de Pitágoras:

$(2)^2 + (\sqrt{5})^2 = (x+1)^2$

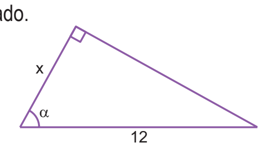
$9 = (x+1)^2$

$3 = x + 1$

$\Rightarrow x = 2$

Clave D

13. Sea α el ángulo cuya secante es igual a 2,4. Del enunciado.



Del triángulo:

$\sec\alpha = \frac{12}{x}$ ... (1)

Por dato:

$\sec\alpha = 2{,}4 = \frac{24}{10} = \frac{12}{5}$ ... (2)

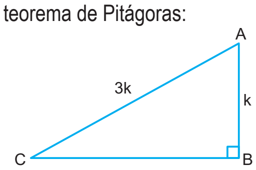
De (2) y (1) $\frac{12}{x} = \frac{12}{5}$

$\therefore x = 5$

Clave A

14. Por dato:

Por el teorema de Pitágoras:

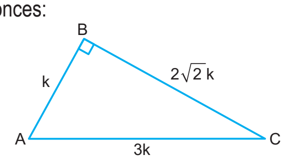


$\overline{CB}^2 + k^2 = (3k)^2$

$\overline{CB}^2 = 8k^2$

$\overline{CB} = 2\sqrt{2}\,k$

Entonces:



Por teorema de correspondencia en el triángulo ABC:

El menor ángulo será el ángulo C.

Nos piden:

$\cos C = \frac{2\sqrt{2}\,k}{3k}$

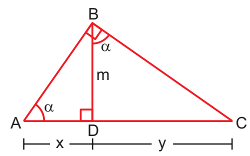
$\therefore \cos C = \frac{2\sqrt{2}}{3}$

Clave B

Nivel 2 (página 20) Unidad 1

Comunicación matemática

15. Del triángulo:



Por dato:

$\frac{AD}{DC} = \frac{x}{y} = \frac{2}{4} = \frac{1}{2}$

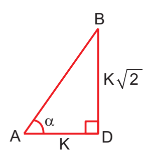
$\Rightarrow x = k;\ y = 2k$

De los triángulos ADB y BDC:

$\tan\alpha = \frac{m}{k} = \frac{2k}{m} \Rightarrow m^2 = 2k^2$

$m = \sqrt{2}\,k$

Reemplazando en el $\triangle$ADB:



Por el teorema de Pitágoras:

$AB^2 = k^2 + (k\sqrt{2})^2$

$AB^2 = 3k^2$

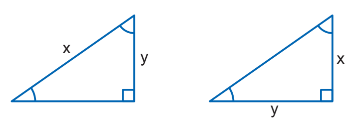
$AB = k\sqrt{3}$

$\sec\alpha = \frac{AB}{AD} = \frac{k\sqrt{3}}{k}$

$\therefore \sec\alpha = \sqrt{3}$

Clave E

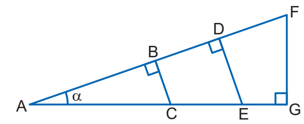
16. En un triángulo rectángulo, sea x, y las longitudes de 2 de sus lados:



I. En cualquier caso, se puede calcular el tercer lado por el teorema de Pitágoras; por lo que se puede calcular cualquiera de las razones trigonométricas de sus ángulos agudos.

... I–V

II. Las razones trigonométricas de un ángulo dependen solo de su amplitud. Sea α ángulo agudo se observa:



Los triángulos ABC, ADE y AGF son semejantes, esto será:

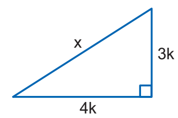
$\triangle ABC \sim \triangle ADE \sim \triangle AGF$

$\Rightarrow \frac{BC}{AC} = \frac{DE}{AE} = \frac{FG}{AF} = \operatorname{sen}\alpha$

$\therefore$ Las razones trigonométricas no dependen de las longitudes de los lados del triángulo.

... II–F

III. En un triángulo rectángulo, por dato:



Por teorema de Pitágoras:

$x^2 = (4k)^2 + (3k)^2$

$x^2 = 16k^2 + 9k^2$

$x^2 = 25k^2$

$x = 5k$

$\therefore$ La hipotenusa es al menor lado como 5 es a 3 respectivamente.

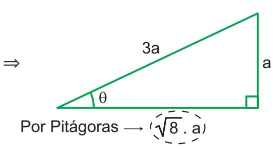
... III–V

Clave E

Razonamiento y demostración

17.

$\operatorname{sen}\theta = \frac{1}{3} \Rightarrow$



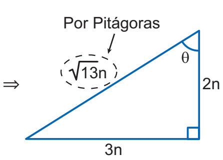
$\Rightarrow \tan\theta = \frac{a}{\sqrt{8}\,a} = \frac{1}{\sqrt{8}}$

Piden: $\tan^2\theta = \left(\frac{1}{\sqrt{8}}\right)^2 = \frac{1}{8}$

Clave E

18.

$\cot\theta = \frac{2}{3} \Rightarrow$



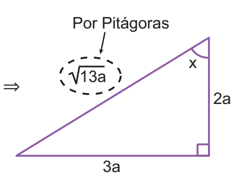
$\Rightarrow \cos\theta = \frac{2n}{\sqrt{13}\,n} = \frac{2}{\sqrt{13}}$

Piden $M = \sqrt{13}\,.\cos\theta = \sqrt{13}\left(\frac{2}{\sqrt{13}}\right) = 2$

Clave B

19.

$\tan x = \frac{3}{2} \Rightarrow$

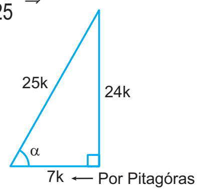


$\Rightarrow \text{sen}x = \dfrac{3a}{\sqrt{13}\,a} = \dfrac{3}{\sqrt{13}}$

Piden $E = \sqrt{13}\,\text{sen}x = \sqrt{13}\left(\dfrac{3}{\sqrt{13}}\right) = 3$

Clave C

20. $\text{sen}\alpha = \dfrac{24}{25} \Rightarrow$

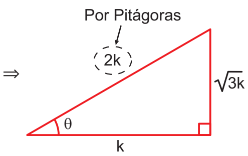


Piden $R = \tan\alpha = \left(\dfrac{24k}{7k}\right) = \dfrac{24}{7}$

Clave E

21.

$\tan\theta = \dfrac{\sqrt{3}}{1} \Rightarrow$

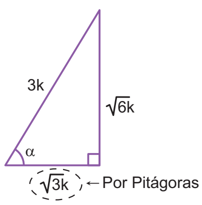


Piden $M = \cos\theta = \left(\dfrac{k}{2k}\right) = \dfrac{1}{2}$

Clave B

22.

$\text{sen}\alpha = \dfrac{\sqrt{6}}{3} \Rightarrow$



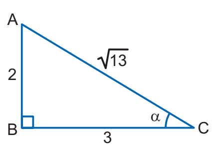
$\Rightarrow \csc\alpha = \dfrac{3k}{\sqrt{6}\,k} = \dfrac{3}{\sqrt{6}}$

Piden $T = \sqrt{6}\csc\alpha + 1 = \sqrt{6}\left(\dfrac{3}{\sqrt{6}}\right) + 1 = 4$

Clave D

23. Dato: $\tan\alpha = \dfrac{2}{3}$

Entonces:



Piden:

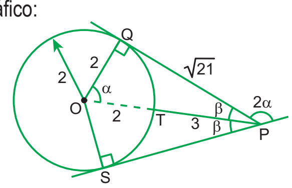
$L = 4\csc^2\alpha - 3$

$L = 4\left(\dfrac{\sqrt{13}}{2}\right)^2 - 3$

$L = 13 - 3 = 10$

Clave E

24. Del gráfico:



$\overline{OP}$ es bisectriz

$OQ = 2$

Por el teorema de Pitágoras: $QP = \sqrt{21}$

Entonces:

$2\beta + 2\alpha = 180°$

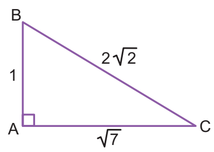
$\beta + \alpha = 90°$

Por lo tanto: $m\angle QOP = \alpha$

Piden: $\tan\alpha = \dfrac{\sqrt{21}}{2}$

Clave D

25. Por el teorema de Pitágoras en el gráfico:

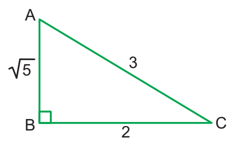


El menor ángulo agudo es C.

$\csc C = \dfrac{2\sqrt{2}}{1} = 2\sqrt{2}$

Clave C

26.

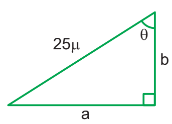


El menor ángulo agudo es A:

$\text{sen}A = \dfrac{2}{3}$

Clave B

27. De los datos; sea θ ángulo de tangente $\dfrac{4}{3}$:



$\tan\theta = \dfrac{a}{b} = \dfrac{4}{3}$

$a = 4k;\ b = 3k$

Por el teorema de Pitágoras:

$a^2 + b^2 = (25)^2$

$(4k)^2 + (3k)^2 = (25)^2$

$25k^2 = (25)^2$

$k^2 = 25$

$k = 5$ ...(1)

Nos piden S, donde:

$S = a + b$

$S = 4k + 3k$

$S = 7k$

De (1)

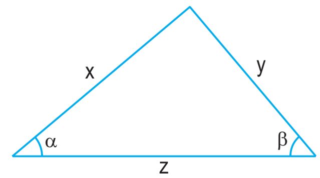
$\therefore\ S = 7 \cdot 5 = 35\ u$

Clave B

Nivel 3 (página 21) Unidad 1

Comunicación matemática

28. Si α y β son complementarios, entonces:

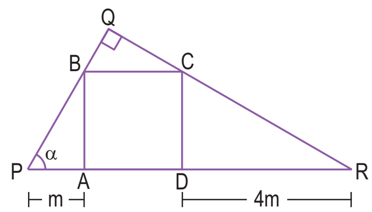


Se cumple el teorema de Pitágoras es decir:

$x^2 + y^2 = z^2$

Clave C

29. Del gráfico, sea x lado del cuadrado ABCD:



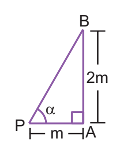
$m\angle RCD = m\angle BPA = \alpha$

De los triángulos BAP y CDR:

$\tan\alpha = \dfrac{x}{m} = \dfrac{4m}{x} \Rightarrow x^2 = 4m^2$

$x = 2m$ ...(1)

En el triángulo rectángulo BAP:



Por el teorema de Pitágoras:

$BC^2 = m^2 + (2m)^2$

$PB^2 = 5\,m^2$

$PB = \sqrt{5}\,m$ ...(2)

I. De (2)

$\text{sen}\alpha = \dfrac{2m}{m\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

$\therefore\ \text{sen}\alpha = \dfrac{2\sqrt{5}}{5}$

... I–F

II. De (1)

$PR = PA + AD + DR$

$PR = m + 2\,m + 4\,m$

$PR = 7\,m$

Luego: $\dfrac{PA}{PR} = \dfrac{m}{7m}$

PA y PR están en razón de 1 a 7 respectivamente.

... II–V

III. De (2)

$\sec\alpha = \dfrac{m\sqrt{5}}{m}$

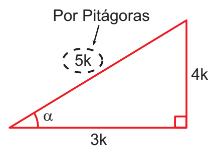
$\therefore\ \sec\alpha = \sqrt{5}$

... III–V

Clave A

Razonamiento y demostración

30. $\cot\alpha = 0{,}75 = \dfrac{3}{4}$



$\Rightarrow \sec\alpha = \dfrac{5k}{3k} = \dfrac{5}{3}$

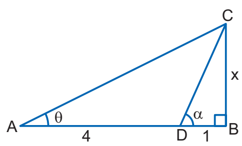
$\Rightarrow \tan\alpha = \dfrac{4k}{3k} = \dfrac{4}{3}$

Piden:

$E = \sec\alpha - \tan\alpha = \left(\dfrac{5}{3}\right) - \left(\dfrac{4}{3}\right) = \dfrac{1}{3}$

Clave E

31. Del gráfico:



Sea: CB = x

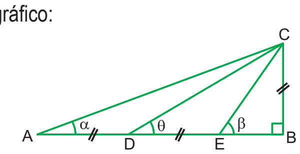
Piden:

$L = \tan\alpha\cot\theta$

$L = \dfrac{x}{1} \cdot \dfrac{5}{x} = 5$

Clave B

32. Del gráfico:



AD = DE = CB

$\cot\alpha = \dfrac{AB}{CB} = \dfrac{2CB + EB}{CB}$

$\cot\beta = \dfrac{EB}{CB}$

$\cot\theta = \dfrac{BD}{CB} = \dfrac{CB + EB}{CB}$

Piden:

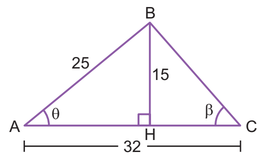
$L = \dfrac{\cot\alpha - \cot\beta}{\cot\theta - \cot\beta}$

$L = \dfrac{\dfrac{2CB + EB}{CB} - \dfrac{EB}{CB}}{\dfrac{CB + EB}{CB} - \dfrac{EB}{CB}} = \dfrac{2CB + EB - EB}{CB + EB - EB}$

$L = \dfrac{2CB}{CB} = 2$

Clave B

33. Del gráfico:



Dato:

$\operatorname{sen}\theta = \dfrac{3}{5} = \dfrac{3x}{5x} = \dfrac{BH}{AB}$

Pero AB = 25

Entonces: $25 = 5x \Rightarrow x = 5$

$\therefore$ BH = 3x = 3(5) = 15

Pero ⊿AHB es notable (37° y 53°), entonces:
AH = 4x = 4(5) = 20
HC = 32 – 20 = 12

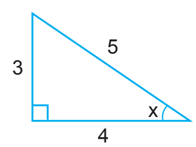
Piden:

$\tan\beta = \dfrac{BH}{HC} = \dfrac{15}{12} = \dfrac{5}{4}$

Clave D

34. Dato: $\operatorname{sen}x = \dfrac{3}{5}$

Entonces:



Piden:

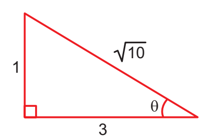
$A = \tan x + \cot x$

$A = \dfrac{3}{4} + \dfrac{4}{3} = \dfrac{25}{12}$

Clave C

35. Dato: $\tan\theta = \dfrac{1}{3}$

Entonces:

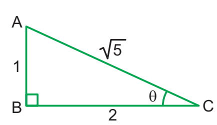


Piden: $B = \operatorname{sen}\theta + \cos\theta$

$B = \dfrac{1}{\sqrt{10}} + \dfrac{3}{\sqrt{10}} = \dfrac{4}{\sqrt{10}} = \dfrac{2\sqrt{10}}{5}$

Clave E

36. Dato: $\cot\theta = 2$
Entonces:



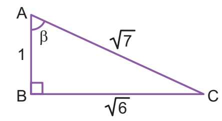
Piden:
$L = 5\operatorname{sen}^2\theta + 4\sec^2\theta$

$L = 5\left(\dfrac{1}{\sqrt{5}}\right)^2 + 4\left(\dfrac{\sqrt{5}}{2}\right)^2$

$L = 1 + 5 = 6$

Clave B

37. Dato: $\sec\beta = \sqrt{7}$



Piden: $L = 6\csc^2\beta + \tan^2\beta$

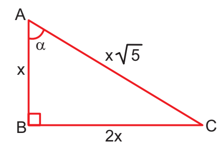
$L = 6\left(\dfrac{\sqrt{7}}{\sqrt{6}}\right)^2 + \left(\dfrac{\sqrt{6}}{1}\right)^2$

$L = 7 + 6 = 13$

Clave E

Resolución de problemas

38.

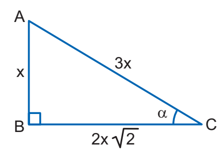


El mayor ángulo agudo es A:

$\csc A = \dfrac{x\sqrt{5}}{2x} = \dfrac{\sqrt{5}}{2}$

Clave B

39.



El menor ángulo agudo es C.

Piden: $L = \operatorname{sen}\alpha\tan\alpha$

$L = \dfrac{x}{3x} \cdot \dfrac{x}{2x\sqrt{2}} = \dfrac{1}{6\sqrt{2}} = \dfrac{\sqrt{2}}{12}$

Clave D

MARATÓN MATEMÁTICA (página 22)

1. Factorizamos la expresión:

$K = \dfrac{1^\circ(1 + 2 + 3 + ... + n)}{1^g(1 + 2 + 3 + ... + n)}$

$K = \dfrac{1^\circ}{1^g};\ 9^\circ = 10^g \Rightarrow 1^\circ = \dfrac{10^g}{9}$

$K = \dfrac{10^g}{9.1^g} = \dfrac{10}{9}$

Clave B

2. Sabemos:
Suma de ángulos internos = 360°
Luego tenemos:

$A + 88^\circ + \dfrac{3}{4}\pi\ \text{rad} + 80^g = 360^\circ$

$A + 88^\circ + \dfrac{3}{4}\pi\left(\dfrac{180^\circ}{\pi}\right) + 80^g\left(\dfrac{9^\circ}{10^g}\right) = 360^\circ$

$A + 88^\circ + 135^\circ + 72^\circ = 360^\circ$

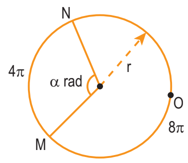
$A = 65^\circ$

$A = 65^\circ\left(\dfrac{\pi}{180^\circ}\right)$

$\therefore\ A = \dfrac{13\pi}{36}\text{rad}$

Clave D

3. Del gráfico tenemos:

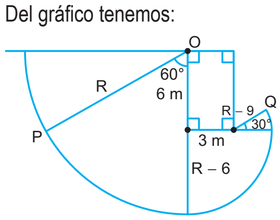


- $4\pi + 8\pi = 2\pi(r)$
  $12\pi = 2\pi(r)$
  $r = 6$

- En $\overset{\frown}{MN}$:
  $4\pi = \alpha(r)$
  $4\pi = 6\alpha$
  $\therefore\ \alpha = \dfrac{2\pi}{3}$

Clave A

4. Del gráfico tenemos:



$$4{,}5\pi = \left(\frac{\pi}{3}\right)R + (R-6)\frac{\pi}{2} + (R-9)\frac{\pi}{6}$$

$$4{,}5\pi = \frac{2\pi R + 3\pi R - 18\pi R + \pi R - 9\pi}{6}$$

$$27\pi = 6\pi R - 27\pi \Rightarrow 54\pi = 6\pi R$$

$$\therefore\ R = 9\text{ m}$$

Clave C

5. Aplicamos el teorema de Pitágoras:

$(5b+2)^2 = (3b-1)^2 + (4b+3)^2$

$25b^2 + 20b + 4 = 9b^2 - 6b + 1 + 16b^2 + 24b + 9$

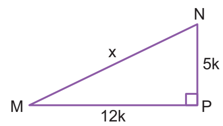
$25b^2 + 20b + 4 = 25b^2 + 18b + 10$

$2b = 6 \Rightarrow b = 3$

Luego: $\cot\theta = \dfrac{4b+3}{3b-1} = \dfrac{15}{8}$

Clave D

6.



Por el teorema de Pitágoras

$x^2 = (5k)^2 + (12k)^2 \Rightarrow x = 13k$

Tenemos como dato:

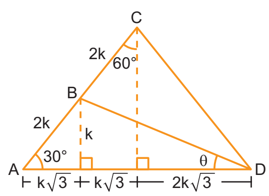
$x + 12k + 5k = 60$

$30k = 60\text{ m} \Rightarrow k = 2\text{ m}$

$\therefore\ x = 26\text{ m}$

Clave B

7. En el gráfico tenemos:



$\cot\theta = \dfrac{3k\sqrt{3}}{k} = 3\sqrt{3}$

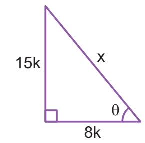
Calculamos:

$M = \dfrac{\sqrt{3}}{3}(\cot\theta) = \dfrac{\sqrt{3}}{3}(3\sqrt{3})$

$\therefore\ M = 3$

Clave D

8. Graficamos al triángulo rectángulo:



Por el teorema de Pitágoras

$x^2 = (15k)^2 + (8k)^2 \Rightarrow x^2 = 289k^2$

$x = 17k$

Calculamos R:

$R = (\operatorname{sen}\theta + 1)\sec\theta = \left(\dfrac{15}{17} + 1\right)\times\dfrac{17}{8}$

$R = \dfrac{32}{17}\times\dfrac{17}{8} = 4$

$\therefore\ R = 4$

Clave A

9.

$1{,}11° = S°\,A'\,N''$

$1° + 0{,}11°\left(\dfrac{60'}{1°}\right) = 1° + 6{,}6' = 1° + 6' + 0{,}6'$

$S°\,A'\,N'' = 1° + 6' + 0{,}6'\left(\dfrac{60''}{1'}\right)$

$S°\,A'\,N'' = 1° + 6' + 36''$

$\Rightarrow S = 1$
$A = 6$
$N = 36$

Luego:

$P = S + A + N = 43$

Clave D

Unidad 2

PROPIEDADES DE LAS RAZONES TRIGONOMÉTRICAS

PRACTIQUEMOS

Nivel 1 (página 27) Unidad 2

Comunicación matemática

1. I. La razón recíproca para el seno del ángulo es la cosecante de dicho ángulo

... (csc)

II. Sean α y β ángulos complementarios luego se cumple:

$$\sec\beta = \csc\alpha$$

... (csc)

III. Para un ángulo la tangente y la cotangente son recíprocas; su producto es la unidad.

... (cot)

Clave C

2. I. Para un ángulo, el seno y la secante no son recíprocas

... (verdadera)

II. Sea 2 ángulos α y θ complementarios:

$\tan\alpha\tan\theta = \tan\alpha\cot\alpha = 1$

... (falsa)

III. Sean β y ω ángulos complementarios

$\tan\beta = \cot\omega$

$\therefore \dfrac{\tan\beta}{\cot\omega} = 1$

... (verdadera)

Clave D

Razonamiento y demostración

3. $\sec(2x - 10°) = \csc 32°$

Se debe cumplir:

$2x - 10° + 32° = 90°$

$2x = 68° \Rightarrow x = 34°$

Clave C

4. $\tan(x + 20°)\cot 80° = 1$

$x + 20° = 80° \Rightarrow x = 60°$

Clave E

5. $E = \dfrac{\text{sen}40° + \cos 50°}{\text{sen}40°}$

$E = \dfrac{\text{sen}40° + \text{sen}40°}{\text{sen}40°}$

$E = \dfrac{2\text{sen}40°}{\text{sen}40°} \quad \therefore E = 2$

Clave A

6. $\text{sen}3x = \cos x$

$3x + x = 90°$

$4x = 90°$

$x = \dfrac{45°}{2}$ a radianes

$\Rightarrow \dfrac{\frac{45}{2}}{9} = \dfrac{R}{\frac{\pi}{20}}$

$\dfrac{5}{2} \cdot \dfrac{\pi}{20} = R \quad \Rightarrow \quad R = \dfrac{\pi}{8}$

$\therefore x = \dfrac{\pi}{8}$ rad

Clave C

7. $\tan 4x \cdot \cot 8y = 1$

$\Rightarrow 4x = 8y$

$x = 2y$

$\therefore \dfrac{x}{y} = 2$

Clave B

8. En la expresión:

$\cos(10° + a) = \text{sen}3a$

$\Rightarrow (10° + a)$ y $3a$ son complementarios:

$10° + a + 3a = 90°$

$4a = 80°$

$a = 20° \cdot \dfrac{\pi\text{rad}}{180°}$

$\therefore a = \dfrac{\pi}{9}$ rad

Clave B

9. De la expresión:

$$\frac{\tan\left(\frac{4\pi}{9} - a\right)}{\cot\left(\frac{\pi}{3} - b\right)} = 1$$

$\tan\left(\dfrac{4\pi}{9} - a\right) = \cot\left(\dfrac{\pi}{3} - b\right)$

$\left(\dfrac{4\pi}{9} - a\right)$ y $\left(\dfrac{\pi}{3} - b\right)$ complementarios

$\dfrac{4\pi}{9} - a + \dfrac{\pi}{3} - b = \dfrac{\pi}{2}$

$a + b = \dfrac{4\pi}{9} + \dfrac{\pi}{3} - \dfrac{\pi}{2}$

$a + b = \dfrac{5\pi}{18}$

$\therefore \dfrac{a + b}{5} = \dfrac{\pi}{18}$rad.

Clave D

10. De la expresión:

$\underbrace{\csc 24°}_{\sec 66°}\cos\alpha = 1$

sec66° (complementarios)

$\sec 66°\cos\alpha = 1$

Por razones trigonométricas recíprocas:

$\therefore \alpha = 66°$

Clave D

Resolución de problemas

11. Del enunciado:

$\tan\beta\underbrace{\tan 19°}_{\cot 71°} = 1$

cot71° (complemento)

$\tan\beta\cot 71° = 1$

Por razones trigonométricas recíprocas

$\beta = 71°$

$\therefore 2\beta = 142°$

Clave C

12. Del enunciado, sea x el valor a agregar y θ ángulo agudo:

$x + \text{sen}\theta\csc\theta = 4$

Por razones trigonométricas recíprocas

$x + 1 = 4$

$\therefore x = 3$

Clave E

13. Del enunciado:

$\dfrac{\tan 46°}{\cot 4\alpha} = 1$

$\tan 46° = \cot 4\alpha$

Por razones de ángulos complementarios:

$46° + 4\alpha = 90°$

$4\alpha = 90° - 46°$

$4\alpha = 44°$

$\alpha = 11°$

$\therefore 6\alpha = 66°$

Clave E

Nivel 2 (página 27) Unidad 2

Comunicación matemática

14. A) $\text{sen}\alpha$ y $\sec\beta$ son recíprocas

si: $\text{sen}\alpha \cdot \underbrace{\sec\beta}_{\csc\alpha} = 1$

$\csc\alpha$ (ya que $\alpha + \beta = 90°$) (V)

B) $\sec\beta = \csc\alpha$ (α y β complementarios) (V)

C) $\tan\beta = \cot\alpha \Rightarrow \tan\beta \neq \tan\alpha$ (F)

D) $\csc\beta = \sec\alpha$; si $\alpha + \beta = 90°$

$\Rightarrow \dfrac{\csc\beta}{\sec\alpha} = 1$ (V)

Clave C

15. En la expresión: A) V B) V C) V D) V

$\Rightarrow \tan\phi \cdot \cot\left(\dfrac{\pi}{2} - \phi\right) = 1$

Por razones trigonométricas recíprocas

$\phi = \dfrac{\pi}{2} - \phi$

$2\phi = \dfrac{\pi}{2}$

$\phi = \dfrac{\pi}{4} = 45°$

Pero $\phi \neq 45°$ (dato)

$\therefore \tan\phi \cdot \cot\left(\dfrac{\pi}{2} - \phi\right) \neq 1$ F

Clave E

Razonamiento y demostración

16. $\tan 5x\cot(x + 12°) = 1$

Se debe cumplir:

$5x = x + 12° \Rightarrow x = 3°$

Clave C

17. $\text{sen}3x\csc(x + 40°) = 1$

Se debe cumplir:

$3x = x + 40° \Rightarrow x = 20°$

Clave B

18. $\cos 2x\sec 70° = 1$

Se debe cumplir:

$2x = 70° \Rightarrow x = 35°$

Clave C

19. $\tan 3x\cot(12° - x) = 1$

Se debe cumplir:

$3x = 12° - x \Rightarrow x = 3°$

Clave C

20. $\text{sen}2x\csc(42° - x) = 1$

Se debe cumplir:

$2x = 42° - x \Rightarrow x = 14°$

Clave C

21. $\tan 3x = \cot(x + 10°)$

Se debe cumplir:

$3x + x + 10° = 90°$

$\Rightarrow x = 20°$

Clave B

22. $\sec 2x = \csc(x + 12°)$

Se debe cumplir:

$2x + x + 12° = 90°$

$3x = 78°$

$\Rightarrow x = 26°$

Clave D

23. $\tan 4x \tan x = 1$

$\tan 4x \cot(90° - x) = 1$

Se debe cumplir:

$4x = 90° - x$

$\Rightarrow x = 18°$

Clave C

24. $\tan 5x \cot(x + 20°) = 1$

Se debe cumplir:

$5x = x + 20°$

$\Rightarrow x = 5°$

Clave A

25. $\tan 3x \cot(x + 10°) = 1$

Se debe cumplir:

$3x = x + 10°$

$\Rightarrow x = 5°$

Clave A

Resolución de problemas

26. Del dato:

$a + c + b - c = \pi/2$ rad

$a + b = \pi/2$

$\Rightarrow$ a y b son complementarios

$\tan a = \cot b$

$$\frac{\tan a}{\cot b} = 1$$

$$\therefore\ 2 \cdot \frac{\tan a}{\cot b} = 2$$

Clave A

27. De los datos:

$3\alpha + 2\beta = 125°$ ... (1)

Además:

$\cos\beta \sec\alpha = 1$

Por razones trigonométricas recíprocas

$\beta = \alpha$

En (1):

$3\alpha + 2\alpha = 125°$

$5\alpha = 125°$

$\alpha = 25°$

Nos piden:

$\alpha + \beta = \alpha + \alpha = 2\alpha = 50°$

$\therefore\ \alpha + \beta = 50°$

Clave E

28. Por dato:

$$\text{sen}(3x - 11)°\underbrace{\sec 38°}_{\csc 52°} = 1$$

$\text{sen}(3x - 11)° \csc 52° = 1$

recíprocos:

$\Rightarrow (3x - 11)° = 52°$

$3x - 11 = 52$

$3x = 63$

$x = 21$

Nos piden:

$$\frac{1}{3} \cdot \frac{\text{sen}(2x + 10)°}{\cos(x + 17)°} = \frac{1}{3} \cdot \frac{\text{sen}(2.21 + 10)°}{\cos(21 + 17)°}$$

$$= \frac{1}{3} \cdot \frac{\text{sen}52°}{\cos 38°}$$

Donde:

$52° + 38° = 90°$

$\Rightarrow \text{sen}52° = \cos 38°$

$$\frac{\text{sen}52°}{\cos 38°} = 1$$

$$\therefore\ \frac{1}{3} \cdot \frac{\text{sen}(2x + 10)°}{\cos(x + 17°)} = \frac{1}{3}$$

Clave B

Nivel 3 (página 28) Unidad 2

Comunicación matemática

29. I. De la proposición

$\text{sen}29°\sec 61° = \text{sen}29°\csc 29°$

complementarios

$\text{sen}29°\sec 61° = 1$

$\therefore$ El sen29° y sec61° son recíprocos

(verdadera)

II. Del enunciado:

$$\frac{\tan\left(\frac{116}{3}\right)^{\circ}}{\tan\left(\frac{154}{3}\right)^{\circ}} = 1$$

$$\Rightarrow \tan\left(\frac{116}{3}\right)^{\circ} = \tan\left(\frac{154}{3}\right)^{\circ}$$

$$\left(\frac{116}{3}\right)^{\circ} + \left(\frac{154}{3}\right)^{\circ} = 90°$$

$\left(\frac{116}{3}\right)^{\circ}$ y $\left(\frac{154}{3}\right)^{\circ}$ complementarios

$$\Rightarrow \tan\left(\frac{116}{3}\right)^{\circ} = \cot\left(\frac{154}{3}\right)^{\circ} \neq \tan\left(\frac{154}{3}\right)^{\circ}$$

(falsa)

III. Se tiene:

$\csc 57°\cos 33° = \csc 57°\text{sen}57° = 1$

complementarios

$\Rightarrow \csc 57°\cos 33° = 1$

$\therefore$ cos33° es el inverso multiplicativo de csc57°, es decir son recíprocos

(verdadera)

Clave D

30. A) De la expresión:

$\text{sen}\alpha - \cos\theta = 0$

$\text{sen}\alpha = \cos\theta$

Por razones trigonométricas de ángulos complementarios:

$\alpha + \theta = \frac{\pi}{2}$ rad

... (falsa)

B) En la expresión:

$$\frac{\tan\alpha - \cot\beta}{\cot\omega} = 0$$

$\Rightarrow \tan\alpha - \cot\beta = 0$

$\tan\alpha = \cot\beta$

$\therefore\ \alpha + \beta = 90°$

... (verdadera)

C) De la igualdad:

$\sec(2x - 18°)\text{sen}30° = 1$

$\sec(2x - 18°)\cos 60° = 1$

Por razones trigonométricas recíprocas:

$2x - 18° = 60° \Rightarrow 2x = 78° \Rightarrow x = 39°$

... (verdadera)

D) $\tan(3x + 15°)\tan 72° = 1$

$\tan(3x + 15°)\cot 18° = 1$

Por razones trigonométricas recíprocas:

$3x + 15° = 18° \Rightarrow 3x = 3° \Rightarrow x = 1°$

... (falsa)

Clave E

Razonamiento y demostración

31. $\text{sen}20° = \cos a \Rightarrow 20° + a = 90° \Rightarrow a = 70°$

$\tan 40° = \cot b \Rightarrow 40° + b = 90° \Rightarrow b = 50°$

Piden: $a + b = 70° + 50° = 120°$

Clave C

32. $\cos 75° = \text{sen}\,a \Rightarrow 75° + a = 90° \Rightarrow a = 15°$

$\cot 89° = \tan b \Rightarrow 89° + b = 90° \Rightarrow b = 1°$

Piden: $a + b = 15° + 1° = 16°$

Clave D

33. $\text{sen}80° = \cos x \Rightarrow 80° + x = 90° \Rightarrow x = 10°$

$\sec 78° = \csc y \Rightarrow 78° + y = 90° \Rightarrow y = 12°$

Piden: $x + y = 10° + 12° = 22°$

Clave B

34. $$M = \frac{\text{sen}17°\csc 17° + \tan 27°\cot 27°}{\cos 54°\sec 54°}$$

$\Rightarrow$ Sabemos que: $\text{sen}\alpha\csc\alpha = 1$

$\cos\beta\sec\beta = 1$

$\tan\theta\cot\theta = 1$

$$M = \frac{1 + 1}{1} = \frac{2}{1} = 2$$

Clave E

35. $$M = \left(\frac{\text{sen}80°}{\cos 10°} + \frac{\tan 40°}{\cot 50°}\right)^{1 + \text{sen}30°\csc 30°}$$

Si: $\alpha + \beta = 90° \Rightarrow \text{sen}\alpha = \cos\beta$

$\tan\alpha = \cot\beta$

Además: $\text{sen}\theta\csc\theta = 1$

Reemplazamos:

$$M = \left(\frac{\text{sen}80°}{\text{sen}80°} + \frac{\tan 40°}{\tan 40°}\right)^{1+1}$$

$\Rightarrow M = (1 + 1)^2 = 2^2 = 4$

Clave B

36. $\tan(8x - 8°) = \cot(x + 8°)$

$\Rightarrow (8x - 8°) + (x + 8°) = 90°$

$9x = 90°$

$x = 10°$

Clave E

37. $E = (3\text{sen}36° + 4\cos54°)\csc36°$

$E = \underbrace{3\text{sen}36°\csc36°}_{1} + 4\cos54°\underbrace{\csc36°}_{\sec54°}$

$E = 3 + 4\underbrace{\cos54°\sec54°}_{1}$

$E = 3 + 4 = 7$

Clave D

Resolución de problemas

38. Por dato, se cumple:

$\tan\alpha\tan\beta\tan\phi = \frac{3}{7}$ ... (1)

Si $\alpha + \beta = 90°$

$\tan\alpha = \cot\beta$ $(\times \tan\beta)$

$\tan\alpha\tan\beta = \cot\beta\tan\beta$

$\tan\alpha\tan\beta = 1$

En (1)

$\tan\alpha\tan\beta\tan\phi = 1\tan\phi = \frac{3}{7}$

$\Rightarrow \tan\phi = \frac{3}{7}$ $(\times \cot\phi)$

$\tan\phi\cot\phi = \frac{3}{7}\cot\phi$

$1 = \frac{3}{7}\cot\phi$

$\therefore \cot\phi = 7/3$

Clave D

39. Del enunciado nos piden:

$\frac{\tan^2\beta + \cot^2\theta}{\tan\beta\cot\theta}$

$\frac{\tan^2\beta + \cot^2\theta}{\tan\beta\cot\theta} = \frac{\tan\beta}{\cot\theta} + \frac{\cot\theta}{\tan\beta}$ ... (1)

Pero; β y θ complementarios:

$\Rightarrow \tan\beta = \cot\theta$

En (1)

$\therefore \frac{\tan^2\beta + \cot^2\theta}{\tan\beta\cot\theta} = 1 + 1 = 2$

Clave A

40. Dados los números a y b, del enunciado:

$a + b = \pi \wedge ab = 2$

$\frac{a + b}{ab} = \frac{\pi}{2}$

$\frac{1}{b} + \frac{1}{a} = \frac{\pi}{2}$ ... (1)

Se pide:

$r = \dfrac{\csc\left(\frac{1}{a}\right)}{\sec\left(\frac{1}{b}\right)}$

De (1):

$\frac{1}{a}\text{rad} + \frac{1}{b}\text{rad} = \frac{\pi}{2}\text{rad}$

Son complementarios:

$\Rightarrow \csc\left(\frac{1}{a}\right) = \sec\left(\frac{1}{b}\right)$

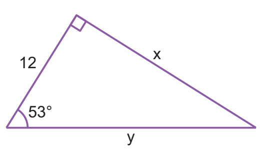
$\therefore r = \dfrac{\csc\left(\frac{1}{a}\right)}{\sec\left(\frac{1}{b}\right)} = 1$

Clave C

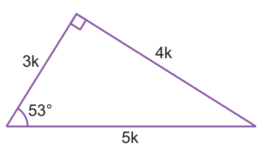
RAZONES TRIGONOMÉTRICAS DE ÁNGULOS NOTABLES

APLICAMOS LO APRENDIDO (página 30) Unidad 2

1.



Notable de 53° y 37°



$12 = 3k$

$k = 4$

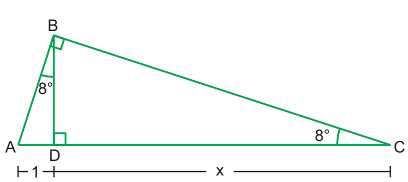
$x = 4k = 4\,.\,4 \quad \wedge \quad y = 5k = 5\,.\,4$

$x = 16 \qquad y = 20$

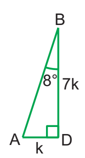
$\therefore x + y = 36$

Clave B

2.



ADB y BDC notables de 8° y 82°

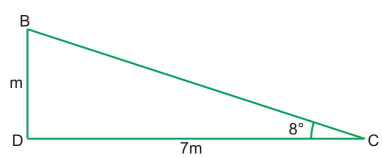


$AD = k = 1$

$\Rightarrow k = 1$

$BD = 7k = 7\,.\,1$

$BD = 7$



$BD = m = 7$

$\Rightarrow m = 7$

$DC = 7m = 7\,.\,7$

$DC = 49$

Luego:

$x + 1 = DC + 1 = 49 + 1$

$\therefore x + 1 = 50$

Clave C

3. $y = \sqrt[3]{19 + 4\sqrt{3}\csc60°}$

$y = \sqrt[3]{19 + 4\sqrt{3}\,.\,\frac{2}{\sqrt{3}}}$

$y = \sqrt[3]{19 + 8}$

$y = \sqrt[3]{27}$ $\quad \therefore y = 3$

Clave A

4. $P = 32[\cos30°\text{sen}45°]^2$

$P = 32\left[\frac{\sqrt{3}}{2}\,.\,\frac{\sqrt{2}}{2}\right]^2$

$P = 32\left[\frac{6}{16}\right]$ $\quad \therefore P = 12$

Clave A

5. $E = (\sec45°\sqrt{2} + 1)^{\sec60°}$

$E = (\sqrt{2}\,.\sqrt{2} + 1)^2$

$E = (3)^2$ $\quad \therefore E = 9$

Clave E

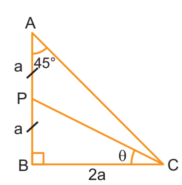
6. $F = 5\text{sen}74° + \sqrt{2}\cos82°$

$F = 5\,.\,\frac{24}{25} + \sqrt{2}\,.\,\frac{1}{5\sqrt{2}}$

$F = \frac{24}{5} + \frac{1}{5}$ $\quad \therefore F = 5$

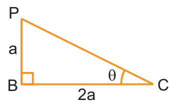
Clave D

7.



ABC notable 45° y 45°

$AB = BC = 2a$
P punto medio de $\overline{AB}$
$PB = a$
Luego:



PBC ◺ notable de $\dfrac{53°}{2}$ y $\dfrac{127°}{2}$

$\therefore\ \theta = \dfrac{53°}{2}$

Clave C

8. Dato:
$\tan(2x + 5°) = \cot(3x + 10°)$

Por razones trigonométricas complementarias:
$2x + 5° + 3x + 10° = 90°$
$5x + 15° = 90°$
$5x = 75°$
$x = 15°$

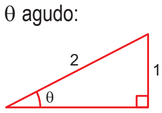
Luego:
$\tan(x + 30°) = \tan(15° + 30°)$
$\tan(x + 30°) = \tan 45°$
$\therefore\ \tan(x + 30°) = 1$

Clave D

9. $\operatorname{sen}\theta = \tan 53°/2$

$\operatorname{sen}\theta = \dfrac{1}{2}$

θ agudo:



◺ notable de 30° y 60°
$\therefore\ \theta = 30°$

Clave A

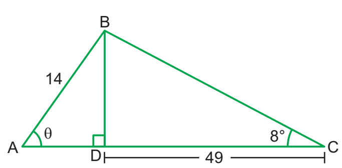
10. Dato:
$\operatorname{sen}3x\csc 48° = 1$

Ángulos recíprocos (3x agudo)
$3x = 48°$
$x = 16°$

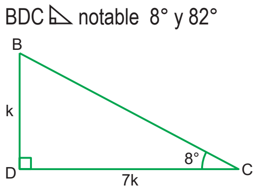
Luego:
$\tan x = \tan 16°$ $\quad\therefore\ \tan x = \dfrac{7}{24}$

Clave C

11.

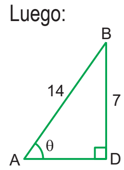


BDC ◺ notable 8° y 82°



$DC = 7k = 49$
$\Rightarrow\ k = 7$
$BD = 7$

Luego:



ADB ◺ notable de 30° y 60°
$\theta = 30°$
$\therefore\ 2\theta = 60°$

Clave A

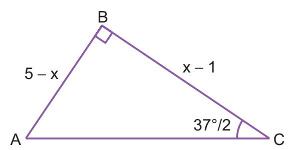
12. $M = \left(\cot\dfrac{53°}{2} + \tan\dfrac{143°}{2}\right)\tan 60°$

$M = (2 + 3)\,.\,\sqrt{3}$

$\therefore\ M = 5\sqrt{3}$

Clave B

13.



◺ ABC notable de $\dfrac{37°}{2}$ y $\dfrac{143°}{2}$

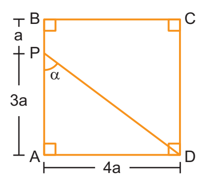
$\tan\dfrac{37°}{2} = \dfrac{5 - x}{x - 1}$

$\dfrac{1}{3} = \dfrac{5 - x}{x - 1}$

$x - 1 = 15 - 3x$
$4x = 16$ $\qquad\therefore\ x = 4$

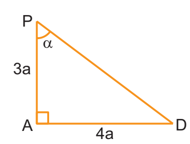
Clave C

14.



Dato:
$\dfrac{AP}{PB} = 3$

$AP = 3PB$; sea $PB = a$
$\Rightarrow AP = 3a$
Como ABCD es un cuadrado:
$AD = AP + PB = 3a + a$
$AD = 4a$



Luego:
PAD ◺ notable 37° y 53°
$\therefore\ \alpha = 53°$

Clave C

PRACTIQUEMOS
Nivel 1 (página 32) Unidad 2

Comunicación matemática

1. I. $\tan\dfrac{37°}{2} = \dfrac{1}{3} \neq \dfrac{1}{2}$

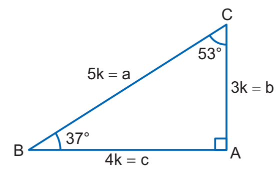
II. $\sec 8° = \dfrac{5\sqrt{2}}{7}$

III. $\csc\dfrac{53°}{2} = \sqrt{5}$

$\therefore$ Solo I es incorrecta.

Clave A

2. BAC ◺ notable de 37° y 53°; luego



- $\dfrac{a}{b} = \dfrac{5k}{3k} = \dfrac{5}{3} \Rightarrow \dfrac{a}{b} = \dfrac{5}{3}$
- $\dfrac{c}{b} = \dfrac{4k}{3k} = \dfrac{4}{3} \Rightarrow \dfrac{c}{b} = \dfrac{4}{3}$
- $\dfrac{a}{c} = \dfrac{5k}{4k} = \dfrac{5}{4} \Rightarrow \dfrac{a}{c} = \dfrac{5}{4}$

$\therefore$ Ib; IIa; IIIc

Clave C

Razonamiento y demostración

3. $M = \sqrt{2\sqrt{3}\operatorname{sen}60° + 6}$

$M = \sqrt{2\sqrt{3}\left(\dfrac{\sqrt{3}}{2}\right) + 6}$

$M = \sqrt{3 + 6} = \sqrt{9} = 3$

Clave C

4. $M = \sqrt{\tan^2 60° + 1}$

$M = \sqrt{(\sqrt{3})^2 + 1}$

$M = \sqrt{3 + 1} = \sqrt{4} = 2$

Clave B

5. $M = \sqrt{12\sec^2 30° + 9}$

$M = \sqrt{12\left(\dfrac{2\sqrt{3}}{3}\right)^2 + 9}$

$M = \sqrt{12\left(\dfrac{4}{3}\right) + 9}$

$M = \sqrt{16 + 9} = \sqrt{25} = 5$

Clave A

6. $A = \sqrt{27\tan^2 53° + 1}$

$A = \sqrt{27\left(\dfrac{4}{3}\right)^2 + 1}$

$A = \sqrt{48 + 1} = \sqrt{49} = 7$

Clave C

7. $y = \sqrt{20\cos^2 30° + 1}$

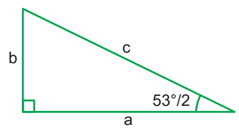
$y = \sqrt{20\left(\dfrac{\sqrt{3}}{2}\right)^2 + 1}$

$y = \sqrt{20\left(\dfrac{3}{4}\right) + 1}$

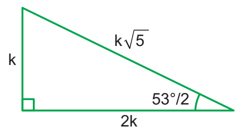
$y = \sqrt{15 + 1} = \sqrt{16} = 4$

Clave D

8.



$\triangle$ notable de $\dfrac{53°}{2}$ y $\dfrac{127°}{2}$



$a = 2k$
$b = k$
$c = k\sqrt{5}$

Luego:

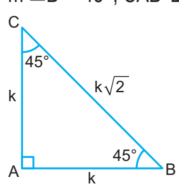
$M = \dfrac{\sqrt{5}\,c + b}{3a}$

$M = \dfrac{\sqrt{5}\,.k\sqrt{5} + k}{3.2k}$

$M = \dfrac{5+1}{6}$ $\therefore\ M = 1$

Clave E

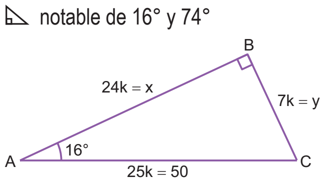
9. $m\angle B = 45°$, CAB $\triangle$ notable de 45°



$BC = k\sqrt{2}$
$24 = k\sqrt{2}$
$k = 12\sqrt{2}$
$AC = k$
$\therefore\ AC = 12\sqrt{2}$

Clave C

10. ABC $\triangle$ notable de 16° y 74°

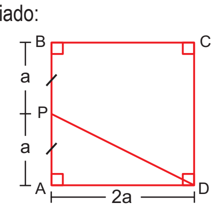


$25k = 50$
$k = 2$
Luego:
$x - y = 24k - 7k$
$x - y = 17k = 17\,.\,2$ $\therefore\ x - y = 34$

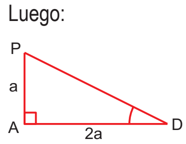
Clave D

Resolución de problemas

11. Del enunciado:



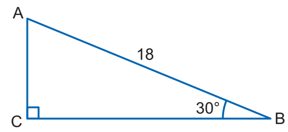
P: punto medio de $\overline{AB}$
$AP = PB = a$
Si ABCD es un cuadrado:
$\Rightarrow AB = AD = 2a$
Luego:



PAD $\triangle$ notable de 53°/2
$\therefore\ m\angle PDA = 53°/2$

Clave D

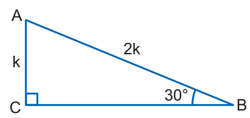
12. Del enunciado:



$m\angle B = \dfrac{90°}{3}$

$m\angle B = 30°$

ACB $\triangle$ notable de 30° y 60°



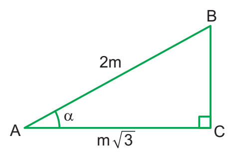
$AB = 2k$
$18 = 2k$
$k = 9$
$AC = k$
$\therefore AC = 9$

Clave B

Nivel 2 (página 32) Unidad 2

Comunicación matemática

13.



ACB $\triangle$ notable de 30° y 60° ... III (V)
$\alpha = 30°$

$\alpha = 30°\ .\ \dfrac{\pi\ \text{rad}}{180°}$

$\alpha = \pi/6$

$\therefore\ \alpha$ es igual a $\dfrac{\pi}{6}$ rad ... I (F)

Luego:

$\dfrac{BC}{AB} = \text{sen}30° = \dfrac{1}{2}$

$AB = 2BC$

$\therefore$ AB es el doble de BC ... II (V)

Clave C

14. I. ABC $\triangle$ notable de 30° y 60° entonces es exacto.
... (correcto)

II. EFD $\triangle$ notable de 37° y 53° de lados 3; 4; 5 entonces EFD es un triángulo pitagórico.
... (correcto)

III. PRQ $\triangle$ notable de 8° y 82° entonces PRQ es un triángulo rectángulo aproximado.
... (correcto)

Clave E

Razonamiento y demostración

15. $A = 10\text{sen}37° + 6\tan53° + \sqrt{2}\sec45°$

$A = 10\left(\dfrac{3}{5}\right) + 6\left(\dfrac{4}{3}\right) + \sqrt{2}\left(\sqrt{2}\right)$

$A = 6 + 8 + 2 = 16$

Clave C

16. $S = \tan60°\csc60° + 3\sqrt{2}\csc45°$

$S = \sqrt{3}\,.\left(\dfrac{2\sqrt{3}}{3}\right) + 3\sqrt{2}\left(\sqrt{2}\right)$

$S = 2 + 6 = 8$

Clave D

17. $A = \sqrt{\sec45°\csc45° + 14\text{sen}30°}$

$A = \sqrt{\left(\sqrt{2}\right)\left(\sqrt{2}\right) + 14\left(\dfrac{1}{2}\right)}$

$A = \sqrt{2+7} = \sqrt{9} = 3$

Clave D

18. $S = \dfrac{\tan53° - \cot53°}{\sec60° + 5}$

$S = \dfrac{\dfrac{4}{3} - \dfrac{3}{4}}{2+5}$

$S = \dfrac{\dfrac{7}{12}}{7} = \dfrac{1}{12}$

Clave D

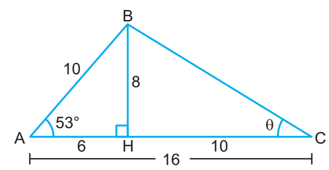
19. $A = \dfrac{\tan^2 60° + \sec^2 45°}{10\text{sen}37° + 4}$

$A = \dfrac{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2}{10\left(\dfrac{3}{5}\right) + 4}$

$A = \dfrac{5}{10} = \dfrac{1}{2}$

Clave B

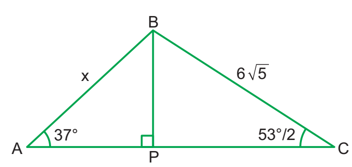
20.



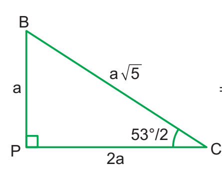
Trazamos la altura BH
AHB $\triangle$ notable 53° y 37°: $AB = 10$
$AH = 6 \wedge BH = 8$
Luego:
$HC = AC - AH$
$HC = 16 - 6$
$HC = 10$
$\therefore\ \cot\theta = \dfrac{10}{8} = \dfrac{5}{4}$

Clave B

21.

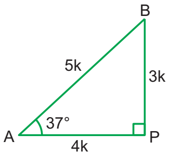


Trazamos $\overline{BP} \perp \overline{AC}$:
BPC $\triangle$ notable de 53°/2



$BC = a\sqrt{5} = 6\sqrt{5}$
$\Rightarrow\ a = 6$
$BP = a$
$BP = 6$

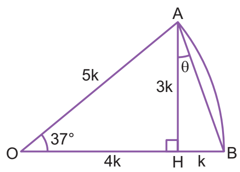
⊿ APB notable de 37° y 53°



BP = 3k = 6
⇒ k = 2
AB = 5k
x = 5 . 2
∴ x = 10

Clave E

22.



AHO ⊿ notable de 37° y 53°
AO = 5k; AH = 3k; OH = 4k
Luego:
AOB sector circular
AO = OB = 5k
HB = OB – OH
HB = 5k – 4k
HB = k

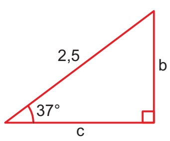
$\cot\theta = \dfrac{AH}{HB} = \dfrac{3k}{k}$

∴ cotθ = 3

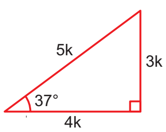
Clave C

Resolución de problemas

23. Del enunciado:



⊿ notable de 37° y 53°



$5k = 2{,}5 = \dfrac{5}{2}$

$k = \dfrac{1}{2}$

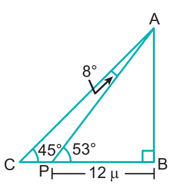
2p: perímetro
2p = 5k + 4k + 3k

$2p = 12k = 12 \cdot \dfrac{1}{2}$

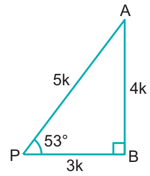
∴ 2p = 6

Clave E

24. Del enunciado:



ABC ⊿ notable de 45°
m∠ACB = 45°
m∠APB = 45° + 8°
m∠APB = 53°
ABP ⊿ notable de 53° y 37°



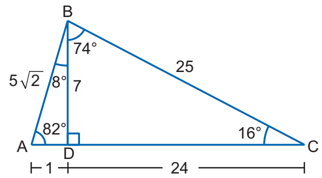
3k = 12
k = 4
AB = 4k = 4 . 4
∴ AB = 16 u

Clave A

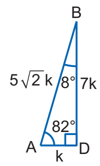
Nivel 3 (página 34) Unidad 2

Comunicación matemática

25.

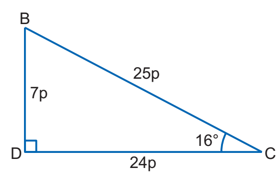


ADB ⊿ notable 8° y 82°



$AB = 5\sqrt{2}\,k$
$5\sqrt{2} = 5\sqrt{2}\,k$
⇒ k = 1
AD = 1 ∧ BD = 7

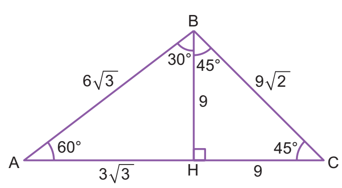
BDC ⊿ notable 16° y 74°



BD = 7p
7 = 7p
p = 1
⇒ DC = 24 ∧ BC = 25
∴ C) DC igual a 20 ... (Incorrecto)

Clave C

26.



BHC ⊿ notable de 45°

$\text{sen}45° = \dfrac{BH}{BC}$

$\dfrac{1}{\sqrt{2}} = \dfrac{BH}{9\sqrt{2}} \Rightarrow BH = 9$

AHB ⊿ notable de 30° y 60° (m∠A = 60°)

$\cos 30° = \dfrac{BH}{AB}$

$\dfrac{\sqrt{3}}{2} = \dfrac{9}{AB} \Rightarrow AB = 6\sqrt{3}$

Luego:

I. tanA es igual a $\sqrt{3}$ ... (verdadero)

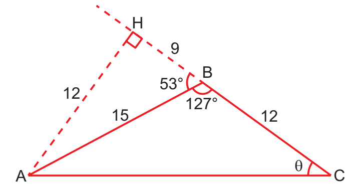
II. AB es igual a $6\sqrt{3}$ ... (verdadero)

III. Altura relativa a $\overline{AC}$ (BH) es igual a 8 ... (falso)

Clave E

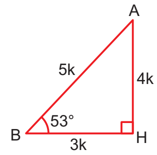
Razonamiento y demostración

27.



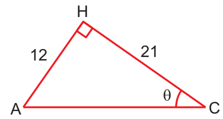
Trazamos la prolongación de BC hasta el punto H donde BH ⊥ AH

Luego:
⊿ AHB notable de 37° y 53°



AB = 5k
15 = 5k
k = 3
HB = 9 AH = 12

En el triángulo rectángulo AHC:



$\tan\theta = \dfrac{12}{21}$

∴ tanθ = 4/7

Clave B

28. Dato:
senxcsc(60° – x) = 1
x agudo, por propiedad de razones trigonométricas recíprocas:
x = 60° – x ⇒ 2x = 60° ⇒ x = 30°
En P:
P = tanxtan2x
P = tan30°tan60°

$P = \dfrac{1}{\sqrt{3}} \cdot \sqrt{3}$ ∴ P = 1

Clave A

29. sec2x = csc4x
Se debe cumplir:
2x + 4x = 90°
6x = 90° ⇒ x = 15°
Piden:
J = cos3xcos4x
J = cos3(15°)cos4(15°)
J = cos45°cos60°

$J = \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{2}}{4}$

Clave D

30. tan5xcot(x + 40°) = 1
Se debe cumplir:
5x = x + 40° ⇒ x = 10°
Piden:

$\text{sen}3x = \text{sen}3(10°) = \text{sen}30° = \dfrac{1}{2}$

Clave B

31. sen2xcsc(x + 40°) = 1

Se debe cumplir:

$2x = x + 40° \Rightarrow x = 40°$

Piden:

$\text{sen}\dfrac{3x}{2} = \text{sen}\dfrac{3(40°)}{2} = \text{sen}60° = \dfrac{\sqrt{3}}{2}$

Clave C

32. sen3x = cos3x

Se debe cumplir:

$3x + 3x = 90° \Rightarrow x = 15°$

Piden:

$\text{sen}2x = \text{sen}2(15°) = \text{sen}30° = \dfrac{1}{2}$

Clave B

33. $M = \cos74°\sec53°\tan\dfrac{127°}{2} - \dfrac{\text{sen}53°}{2}$

$M = \dfrac{7}{25}\cdot\dfrac{5}{3}\cdot 3 - \dfrac{\frac{4}{5}}{2}$

$M = \dfrac{7}{5} - \dfrac{2}{5} \qquad \therefore M = 1$

Clave B

34. $K = \sqrt{40\sec37° + 6\sec53° + 4\cot45°}$

$K = \sqrt{40\left(\dfrac{5}{4}\right) + 6\left(\dfrac{5}{3}\right) + 4(1)}$

$K = \sqrt{50 + 10 + 4}$

$K = \sqrt{64} = 8$

Clave B

35. $S = \sec^2 45° + 5\cos^2 82° + \text{sen}^2\dfrac{37°}{2} - \text{sen}^2\dfrac{53°}{2}$

$S = (\sqrt{2})^2 + 5\left(\dfrac{1}{5\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{10}}\right)^2 - \left(\dfrac{1}{\sqrt{5}}\right)^2$

$S = 2 + \dfrac{5}{50} + \dfrac{1}{10} - \dfrac{1}{5}$

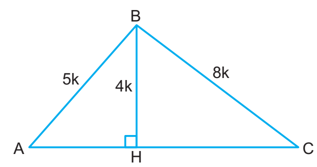
$S = 2 + \dfrac{2}{10} - \dfrac{1}{5}$

$S = 2 + \dfrac{1}{5} - \dfrac{1}{5} \qquad \therefore S = 2$

Clave D

Resolución de problemas

36. Del enunciado:



$\dfrac{AB}{5} = \dfrac{BH}{4} = \dfrac{BC}{8} = k$

Luego:

AHB $\triangle$ notable 37° y 53° $\Rightarrow m\angle A = 53°$

BHC $\triangle$ notable 30° y 60° $\Rightarrow m\angle C = 30°$

$\therefore m\angle A + m\angle C = 53° + 30° = 83°$

Clave E

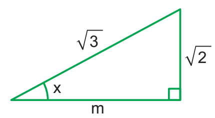
37. Del enunciado, sea x el ángulo agudo:

senx = tan30° . sec45°

$\text{sen}x = \dfrac{1}{\sqrt{3}}.\sqrt{2}$

$\text{sen}x = \dfrac{\sqrt{2}}{\sqrt{3}}$

x agudo:



Del T. de Pitágoras

$m^2 + (\sqrt{2})^2 = (\sqrt{3})^2$

$m^2 + 2 = 3$

$m^2 = 1$

$m = 1$

$\cot x = \dfrac{m}{\sqrt{2}}$

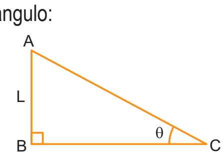
$\therefore \cot x = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Clave D

RESOLUCIÓN DE TRIÁNGULOS RECTÁNGULOS

APLICAMOS LO APRENDIDO (página 35) Unidad 2

1. Sea el triángulo:



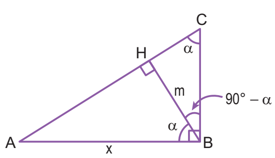
Conocidos el ángulo y el cateto opuesto al ángulo, entonces:

Piden AC:

AC = Lcscθ

Clave D

2. En el gráfico:



Sea $m\angle CBH = 90 - \alpha$

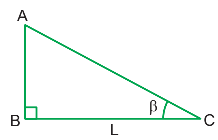
Entonces $m\angle ABH = \alpha$

Por lo tanto:

AB = x = msecα

Clave C

3. Sea el triángulo rectángulo:



Conocidos el ángulo y el cateto adyacente al ángulo, entonces:

AB = Ltanβ

Piden:

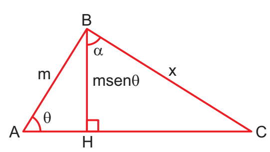
$A_{\triangle ABC} = \dfrac{AB \times BC}{2}$

$A_{\triangle ABC} = \dfrac{L\tan\beta \times L}{2}$

$A_{\triangle ABC} = \dfrac{L^2\tan\beta}{2}$

Clave D

4. En el gráfico:



En el $\triangle$AHB, conocidos el ángulo y la hipotenusa, entonces:

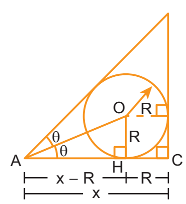
BH = msenθ

En el $\triangle$BHC, conocidos el ángulo y el cateto adyacente al ángulo:

x = msenθsecα

Clave B

5. Del gráfico:



La línea trazada desde el $\angle A$ al centro de la circunferencia es bisectriz, entonces:

$m\angle OAH = \theta$

Además: $OH = R = HC \Rightarrow AH = x - R$

Por lo tanto:

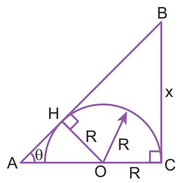
$x - R = R\cot\theta$

$x = R + R\cot\theta = R(1 + \cot\theta)$

$\therefore x = R(\cot\theta + 1)$

Clave C

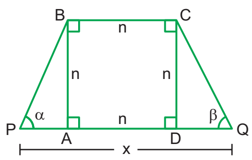
6. Del gráfico:



Trazamos $\overline{OH}$ al punto de tangencia H.
Entonces:
OH = R
Ahora, en el ⊿AHO:
OA = Rcscθ
En el ⊿ACB:
AC = Rcscθ + R
Entonces:
x = (Rcscθ + R)tanθ
x = R(cscθ + 1)tanθ

Clave B

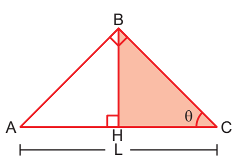
7.



Del gráfico; en el ⊿PAB:
PA = ncotα
En el ⊿CDQ:
DQ = ncotβ
Piden x:
x = PA + AD + DQ
x = ncotα + n + ncotβ
x = n(cotα + cotβ + 1)

Clave B

8. En el gráfico:



En el ⊿ABC:
BC = Lcosθ
En el ⊿BHC:
BH = Lcosθsenθ
HC = Lcosθcosθ = $L\cos^2\theta$

Piden el área sombreada:

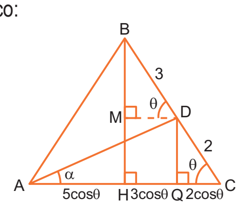
$$S_{somb.} = \frac{HC.BH}{2}$$

$$S_{somb.} = \frac{L\cos^2\theta.L\cos\theta\operatorname{sen}\theta}{2}$$

$$S_{somb.} = \frac{L^2}{2}\cos^3\theta\operatorname{sen}\theta$$

Clave E

9. Del gráfico:

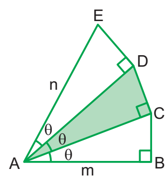


Dato: AB = BC ⇒ $\overline{BH}$ es también mediana.
En el ⊿BHC:
HC = 5cosθ
En el ⊿DQC:
QC = 2cosθ ⇒ HQ = MD = 3cosθ
Además DQ = 2senθ
Entonces en el ⊿AQD:

$$\tan\alpha = \frac{DQ}{AQ} = \frac{2\operatorname{sen}\theta}{8\cos\theta} = 0{,}25\tan\theta$$

Clave C

10. Del gráfico:



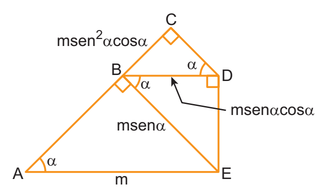
AC = msecθ
AD = ncosθ
DC = ncosθsenθ
Piden el área sombreada:

$$A_{somb.} = \frac{DC \times AC}{2} = \frac{n\cos\theta\operatorname{sen}\theta(m\sec\theta)}{2}$$

$$A_{somb.} = \frac{m\operatorname{sen}\theta\cos\theta}{2\cos\theta} = \frac{mn}{2}\operatorname{sen}\theta$$

Clave C

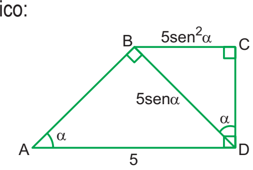
11. Del gráfico:



- ⊿ABE: BE = AEsenα
  = msenα
- ⊿BDE BD = BEcosα
  = msenαcosα
- ⊿BCD BC = BDsenα
  = (msenαcosα)senα
  = $m\operatorname{sen}^2\alpha\cos\alpha$

Clave D

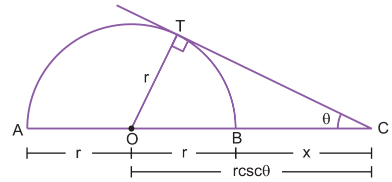
12. Del gráfico:



- ⊿ABD: BD = ADsenα
  BD = 5senα
- ⊿BCD: BC = BDsenα
  BC = (5senα)senα
  = $5\operatorname{sen}^2\alpha$

Clave A

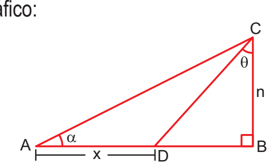
13. Del gráfico:



⊿OTC: OC = OTcscθ
r + x = rcscθ
x = r(cscθ − 1)

Clave B

14. Del gráfico:



En el ⊿ABC:
AB = ncotα
Entonces:
DB = ncotα − x ...(1)

En el ⊿DBC:
DB = ntanθ ...(2)
(1) = (2):
ntanθ = ncotα − x
x = ncotα − ntanθ
x = n(cotα − tanθ)

Clave C

PRACTIQUEMOS

Nivel 1 (página 37) Unidad 2

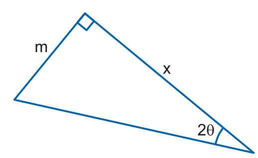
Comunicación matemática

1.

2.

Razonamiento y demostración

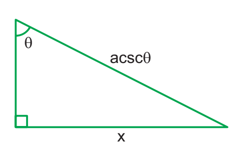
3. Del gráfico, conocidos un ángulo agudo y su cateto opuesto:



x = mcot2θ

Clave D

4. Del gráfico, conocidos un ángulo agudo y la hipotenusa:



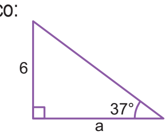
x = acscθsenθ

$$x = a\frac{1}{\operatorname{sen}\theta}\ .\ \operatorname{sen}\theta$$

x = a

Clave D

5. En el gráfico:



Conocidos un ángulo agudo y su cateto opuesto:

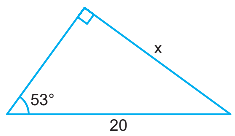
$a = 6\cot 37°$

$a = 6\left(\frac{4}{3}\right)$

$a = 8$

Clave B

6. En el gráfico:



Conocidos un ángulo agudo y la hipotenusa:

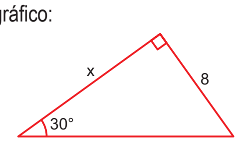
$x = 20\,\text{sen}53°$

$x = 20\left(\frac{4}{5}\right)$

$x = 16$

Clave C

7. En el gráfico:



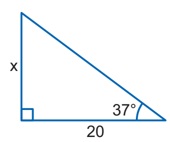
Conocidos un ángulo agudo y su cateto opuesto:

$x = 8\cot 30°$

$x = 8\sqrt{3}$

Clave A

8. En el gráfico:



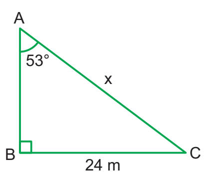
Conocidos un ángulo agudo y su cateto adyacente:

$x = 20\tan 37° = 20\left(\frac{3}{4}\right) = 15$

Clave A

Resolución de problemas

9.

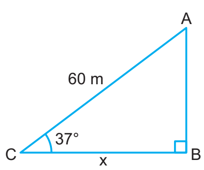


$\Rightarrow x = 24\csc 53°$

$\therefore\ x = 30$ m

Clave D

10.



$\Rightarrow x = 60\cos 37°$

$\therefore\ x = 48$ m

Clave E

Nivel 2 (página 38) Unidad 2

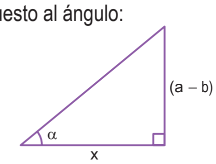
Comunicación matemática

11.

12.

Razonamiento y demostración

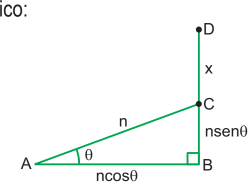
13. Del gráfico, conocidos un ángulo agudo y su cateto opuesto al ángulo:



$x = (a - b)\cot\alpha$

Clave D

14. Del gráfico:



En el ⊿ ABC:

$CB = n\,\text{sen}\theta$

$AB = n\cos\theta$

Pero, por dato:

$DB = AB$

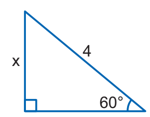
$x + n\,\text{sen}\theta = n\cos\theta$

$x = n\cos\theta - n\,\text{sen}\theta$

$x = n(\cos\theta - \text{sen}\theta)$

Clave D

15. En el gráfico:



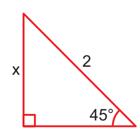
Conocidos un ángulo agudo y la hipotenusa:

$x = 4\,\text{sen}60°$

$x = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

Clave D

16. En el gráfico:



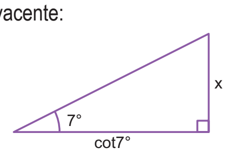
Conocidos un ángulo agudo y la hipotenusa:

$x = 2\,\text{sen}45°$

$x = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

Clave C

17. Del gráfico, conocidos un ángulo agudo y su cateto adyacente:

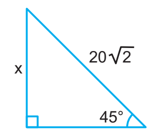


$x = \cot 7°\tan 7°$

$x = 1$

Clave E

18. En el gráfico:



Conocidos un ángulo agudo y la hipotenusa:

$x = 20\sqrt{2}\,\text{sen}45°$

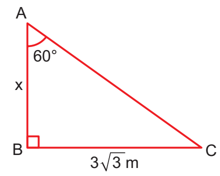
$x = 20\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)$

$x = 20$

Clave B

Resolución de problemas

19.

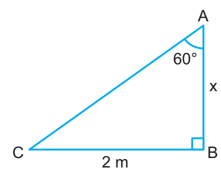


$\Rightarrow x = 3\sqrt{3}\ \cot 60°$

$\therefore\ x = 3$ m

Clave C

20.



$\Rightarrow x = 2\cot 60°$

$\therefore\ x = \frac{2}{3}\sqrt{3}$ m

Clave C

Nivel 3 (página 39) Unidad 2

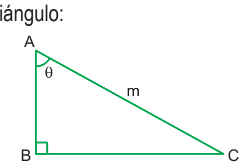
Comunicación matemática

21.

22.

Razonamiento y demostración

23. Sea el triángulo:



$AB = m\cos\theta$

$BC = m\,\text{sen}\theta$

Piden el perímetro del ⊿ABC:

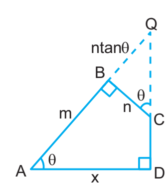
$2p = AB + BC + AC$

$2p = m\cos\theta + m\,\text{sen}\theta + m$

$2p = m(\cos\theta + \text{sen}\theta + 1)$

Clave B

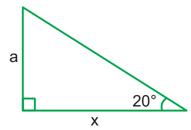
24. Del gráfico:



Trazamos el ⊿ ADQ:
Por cuadrilátero inscriptible:
$m\angle A = m\angle BCQ = \theta$
Entonces: $BQ = n\tan\theta$
En el ⊿ ADQ:
$x = (m + n\tan\theta)\cos\theta$
$x = m\cos\theta + n\,\mathrm{sen}\theta$

Clave B

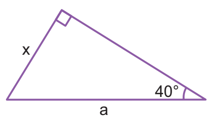
25. Del gráfico, conocidos un ángulo agudo y su cateto opuesto:



$x = a\cot 20°$

Clave D

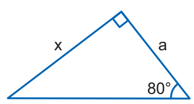
26. Conocidos un ángulo agudo y la hipotenusa:



$x = a\,\mathrm{sen}40°$

Clave A

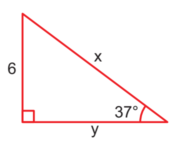
27. Conocidos el ángulo y el cateto adyacente al ángulo:



$x = a\tan 80°$

Clave D

28. En el gráfico:



$x = 6\csc 37° = 6\left(\frac{5}{3}\right) = 10$

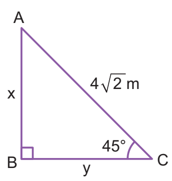
$y = 6\cot 37° = 6\left(\frac{4}{3}\right) = 8$

Piden el perímetro del triángulo:
$6 + x + y = 6 + 10 + 8 = 24$

Clave B

Resolución de problemas

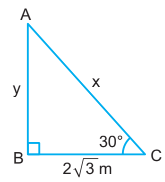
29.



- $x = 4\sqrt{2}\ \mathrm{sen}45°$
  $x = 4$ m
- $y = 4\sqrt{2}\ \cos 45°$
  $y = 4$ m
  $\therefore\ x + y = 8$ m

Clave D

30.



- $y = 2\ \tan 30°$
  $y = 2$ m
- $x = 2\ \sec 30°$
  $x = 4$ m
  $\therefore\ x + y = 6$ m

Clave B

MARATÓN MATEMÁTICA (página 40)

1. De la condición, tenemos:
$\cos(2x + y)\,.\,\csc(x + 3y) = 1$
$\cos(2x + y) = \mathrm{sen}(x + 3y)$
$\Rightarrow\ 2x + y + x + 3y = 90°$
$3x + 4y = 90°$
3x y 4y son complementarios.
$\Rightarrow \tan 3x = \cot 4y$

Nos piden:

$M = \frac{\tan 3x}{\cot 4y}$

$M = \frac{\cot 4y}{\cot 4y} = 1 \qquad \therefore\ M = 1$

Clave E

2. De la condición:
$\mathrm{sen}(8x + 3y)\sec(5y - 2x) - 1 = 0$
$\mathrm{sen}(8x + 3y)\sec(5y - 2x) = 1$
$\mathrm{sen}(8x + 3y) = \cos(5y - 2x)$
$\Rightarrow\ 8x + 3y + 5y - 2x = 90°$
$6x + 8y = 90°$
$3x + 4y = 45°$
Nos piden:
$\tan(4y + 3x) = \tan(45°) = 1$

Clave B

3. Por el teorema de Pitágoras:
$a^2 + b^2 = c^2$
De la condición
$a^2 + b^2 + c^2 + 2ab + 2(a + b)c = 4ab$
$c^2 + c^2 + 2(a + b)c = 2ab$
$2c^2 + 2c(a + b) = 2ab$
$2c(a + b + c) = 2ab$
$2c = \sqrt{2ab}$
$2c = \sqrt{ab}$
$2 = \sqrt{\frac{a}{c}\cdot\frac{b}{c}}$

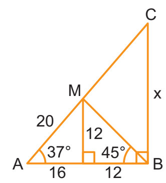
$\therefore\quad 2 = \sqrt{\mathrm{sen}\theta\,.\cos\theta}$

Clave C

4. Por teorema de Pitágoras:
$a^2 + b^2 = c^2$
De la condición tenemos:
$a + b = 2c$
$(a + b)^2 = (2c)^2$
$a^2 + b^2 + 2ab = 4c^2\ \Rightarrow\ c^2 + 2ab = 4c^2$
$2ab = 3c^2$
$2\left(\frac{a}{c}\cdot\frac{b}{c}\right) = 3$
$\therefore\ 2\,\mathrm{sen}\theta\,.\,\cos\theta = 3$

Clave D

5.



- $\tan 37° = \frac{3}{4} = \frac{x}{28}\ \Rightarrow\ x = 21$

Clave C

6. $\cos 60°\sec\theta\tan 23° - \cos^2 45°\csc 30°\cot 67° = \tan 23°$

$\frac{1}{2}\sec\theta\tan 23° - \left(\frac{\sqrt{2}}{2}\right)^2(2)\tan 23° = \tan 23°$

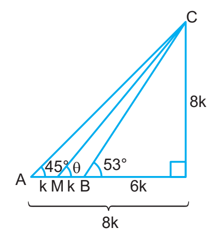
$\frac{\sec\theta}{2} - 1 = 1$

$\sec\theta = 4$

$\therefore\ \cos\theta = \frac{1}{4}$

Clave A

7.



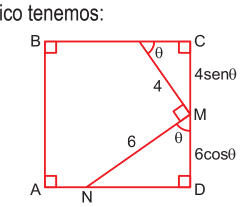
$\tan\theta = \frac{8k}{7k}$

$\tan\theta = \frac{8}{7}$

Nos piden.
$7\tan\theta = 7\left(\frac{8}{7}\right) \qquad \therefore\ 7\tan\theta = 8$

Clave A

8. Del gráfico tenemos:

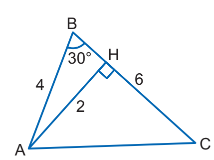


Por dato:
$MC = MD$
$4\,\mathrm{sen}\theta = 6\cos\theta\ \Rightarrow\ \cot\theta = \frac{4}{6}$

$\therefore\ \cot\theta = \frac{2}{3}$

Clave E

9. Del gráfico tenemos:



$A_{\triangle ABC} = 2 \times 6\left(\frac{1}{2}\right) = 6\ u^2$

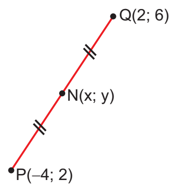
Clave D

Unidad 3

SISTEMA DE COORDENADAS RECTANGULARES

APLICAMOS LO APRENDIDO (página 43) Unidad 3

1.



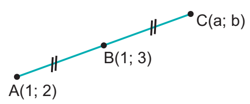
$x = \dfrac{2+(-4)}{2} = -1$

$y = \dfrac{6+2}{2} = 4$

$\therefore N(-1; 4)$

Clave A

2.



$1 = \dfrac{1+a}{2} \Rightarrow a = 1$

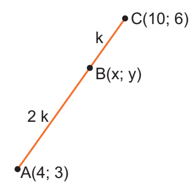
$3 = \dfrac{2+b}{2} \Rightarrow b = 4$

Piden: $a + b = 1 + 4 = 5$

Clave A

3.

AC = 3k
AB = 2k
$\Rightarrow$ BC = k



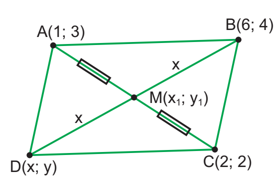
$x = \dfrac{(2k)(10)+(1k)4}{3k} = \dfrac{24k}{3k} = 8$

$y = \dfrac{(2k)6+(k)3}{3k} = \dfrac{15k}{3k} = 5$

$\therefore B(8; 5)$

Clave D

4.



M es punto medio de AC:

$x_1 = \dfrac{1+2}{2} = \dfrac{3}{2}$

$y_1 = \dfrac{3+2}{2} = \dfrac{5}{2}$

También M es punto medio de BD:

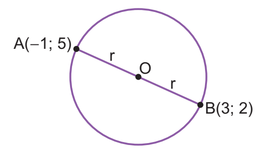
$x_1 = \dfrac{6+x}{2} \Rightarrow \dfrac{3}{2} = \dfrac{6+x}{2}$
$x = -3$

$y_1 = \dfrac{4+y}{2} \Rightarrow \dfrac{5}{2} = \dfrac{4+y}{2}$
$y = 1$

$\therefore$ El punto D tiene coordenadas (–3; 1).

Clave C

5.



Se sabe: $L_{\odot} = 2\pi r$...(I)
Usando la fórmula de la distancia:

$(2r)^2 = (-1-3)^2 + (5-2)^2$
$4r^2 = (-4)^2 + (3)^2$
$4r^2 = 16 + 9$
$4r^2 = 25 \Rightarrow r = \dfrac{5}{2}$

Reemplazando en (I):

$L_{\odot} = 2\pi\left(\dfrac{5}{2}\right) = 5\pi$

Clave B

6.

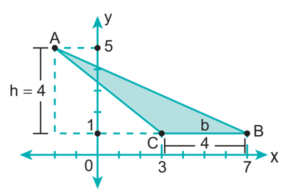
$$\begin{array}{rc|rr|cr} & & -2 & -3 & & \\ -9 & \leftarrow & 3 & 1 & \rightarrow & -2 \\ -1 & \leftarrow & -1 & 4 & \rightarrow & 12 \\ -8 & \leftarrow & -2 & -3 & \rightarrow & 3 \\ \hline -18 & & & & & 13 \end{array}$$

$S_{\Delta} = \dfrac{|13-(-18)|}{2}$

$S_{\Delta} = \dfrac{|13+18|}{2} = \dfrac{31}{2}$ $\quad \therefore S_{\Delta} = 15{,}5$

Clave E

7.

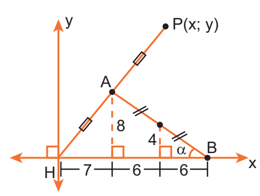


Área del triángulo ABC = $\dfrac{b.h}{2}$

$S_{ABC} = \dfrac{4.4}{2} = 8$ $\quad \therefore S_{ABC} = 8$

Clave D

8.



Por dato: $\tan\alpha = \dfrac{2}{3}$

Del gráfico: el punto H es el origen (0; 0).
El punto A es A(7; 8)

$\Rightarrow$ A es punto medio de HP.

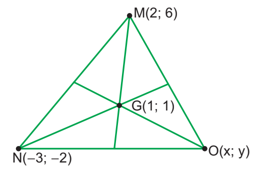
$7 = \dfrac{0+x}{2} \Rightarrow x = 14$

$8 = \dfrac{0+y}{2} \Rightarrow y = 16$

$\therefore P(x; y) = P(14; 16)$

Clave B

9.



$1 = \dfrac{2-3+x}{3}$

$\Rightarrow x = 4$

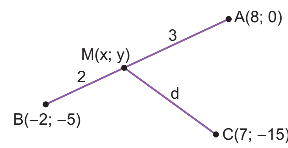
$1 = \dfrac{6-2+y}{3}$

$\Rightarrow y = -1$

$\therefore O(4; -1)$

Clave D

10.



$x = \dfrac{(8)(2)+(-2)(3)}{2+3} = 2$

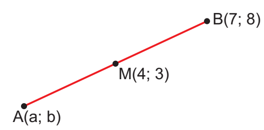
$y = \dfrac{(0)(2)+(-5)(3)}{2+3} = -3$

$\Rightarrow M(x; y) = M(2; -3)$

$d^2 = (7-x)^2 + (-15-y)^2$
$d^2 = (7-2)^2 + (-15-(-3))^2$
$d^2 = (5)^2 + (-12)^2 = 25 + 144$
$d^2 = 169 \Rightarrow d = \sqrt{169}$
$\therefore d = 13$

Clave E

11.



$4 = \dfrac{7+a}{2} \Rightarrow a = 1$

$3 = \dfrac{8+b}{2} \Rightarrow b = -2$

$\therefore A(a; b) = A(1; -2)$

Clave B

12. Hallamos la medida del radio vector:

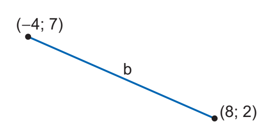
$OP = \sqrt{4^2+(-3)^2}$
$d = \sqrt{16+9} \Rightarrow d = 5$

El área es

$\Rightarrow \pi.r^2 = \pi d^2 = \pi(5)^2$
$\therefore A_O = 25\pi$

Clave A

13.



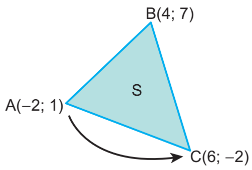
$b = \sqrt{(-4-8)^2 + (7-2)^2}$

$b = \sqrt{(-12)^2 + (5)^2} = \sqrt{169}$

$\therefore b = 13$

Clave C

14. Por dato:



Piden: el área (S) de la región triangular.

$$(+)\ \begin{array}{r|rr|r} & -2 & 1 & \\ 6 & 6 & -2 & 4 \\ -8 & 4 & 7 & 42 \\ -14 & -2 & 1 & 4 \\ \hline -16 & & & 50 \end{array}\ (+)$$

$\Rightarrow S = \dfrac{|50-(-16)|}{2} = \dfrac{|66|}{2} = \dfrac{66}{2}$

$\therefore\ S = 33$

Clave E

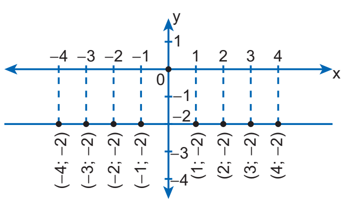
PRACTIQUEMOS

Nivel 1 (página 45) Unidad 3

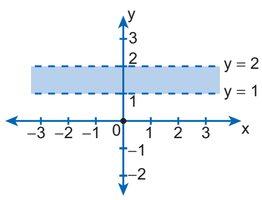
Comunicación matemática

1. Unimos todos los puntos con ordenada –2 y obtenemos una recta.

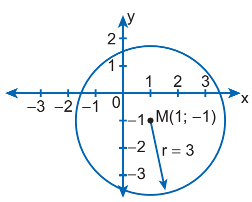
a)



b)



c)



2. Hallamos el lado del cuadrado.

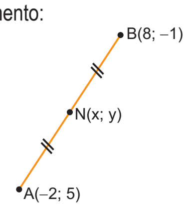
$d = \sqrt{(m-2-(m+1))^2 + (n+3-(n-1))^2}$

$d = \sqrt{(m-2-m-1)^2 + (n+3-n+1)^2}$

$d = \sqrt{(-3)^2 + 4^2} \Rightarrow d = 5;\ M = 4d$

$\therefore\ M = 20$

Sea el segmento:



$x = \dfrac{8+(-2)}{2} = 3$

$y = \dfrac{-1+5}{2} = 2$

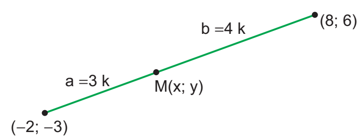
$\therefore\ x + y = 5$

$N = 5\ \Rightarrow\ M = 4N$

Clave B

Razonamiento y demostración

3.



$x = \dfrac{8(3k)+(-2)(4k)}{3k+4k} = \dfrac{16k}{7k}$

$x = \dfrac{16}{7}$

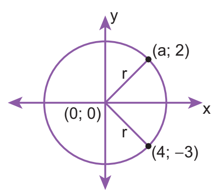
$y = \dfrac{6(3k)+(-3)(4k)}{3k+4k} = \dfrac{6k}{7k}$

$y = \dfrac{6}{7}$

Piden: $x + y = \dfrac{16}{7} + \dfrac{6}{7} = \dfrac{22}{7}$

Clave C

4.



$\Rightarrow r^2 = (4-0)^2 + (-3-0)^2$

$r^2 = 16 + 9 = 25$

$r = 5$

$\Rightarrow r^2 = (a-0)^2 + (2-0)^2$

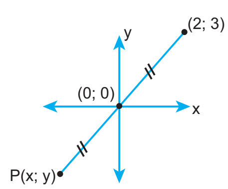
$5^2 = a^2 + 4$

$25 = a^2 + 4$

$21 = a^2 \Rightarrow\ a = \sqrt{21}$

Clave D

5.



$0 = \dfrac{x+2}{2}$

$x = -2$

$0 = \dfrac{y+3}{2}$

$y = -3$

$\therefore P(x; y) = P(-2; -3)$

Clave E

6. Un punto en el eje x, tiene la forma: (x; 0)

$\Rightarrow 5^2 = (x-2)^2 + (0-4)^2$

$25 = (x-2)^2 + 16$

$9 = (x-2)^2$

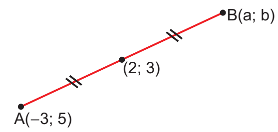
$(x-2) = \pm 3 \Rightarrow\quad x - 2 = 3\ \vee\ x - 2 = -3$

$x = 5\ \vee\quad x = -1$

$\therefore$ El punto puede ser: (5; 0) $\vee$ (–1; 0)

Clave E

7.



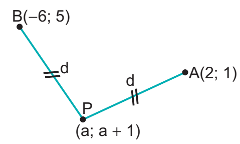
$2 = \dfrac{-3+a}{2} \Rightarrow a = 7$

$3 = \dfrac{5+b}{2} \Rightarrow b = 1$

Piden: $a + b = 7 + 1 = 8$

Clave D

8.



$d^2 = [a-(-6)]^2 + (a+1-5)^2$

$= (a+6)^2 + (a-4)^2 \quad$ ...(I)

$d^2 = (a-2)^2 + (a+1-1)^2$

$= (a-2)^2 + (a)^2 \quad$ ...(II)

De (I) y (II):

$\underbrace{(a+6)^2 + (a-4)^2}_{2a^2+4a+52} = \underbrace{(a-2)^2 + a^2}_{2a^2-4a+4}$

$2a^2 + 4a + 52 = 2a^2 - 4a + 4$

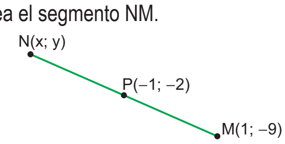
$8a = -48$

$\therefore a = -6$

Clave B

Resolución de problemas

9. • Sea el segmento NM.



• Sabemos:

$P = \left(\dfrac{x_1+x_2}{2}; \dfrac{y_1+y_2}{2}\right)$

• Reemplazamos:

$(-1; -2) = \left(\dfrac{x+1}{2}; \dfrac{y-9}{2}\right)$

• Entonces:

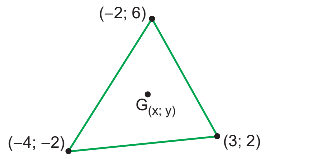
$-1 = \dfrac{x+1}{2} \Rightarrow x = -3$

$-2 = \dfrac{y-9}{2} \Rightarrow y = 5$

$\therefore$ N(–3; 5)

Clave D

10. Sea el baricentro = G(x; y), entonces:



$$x = \frac{-2+3+(-4)}{3} = \frac{-3}{3} = -1$$

$$y = \frac{6+2+(-2)}{3} = \frac{6}{3} = 2$$

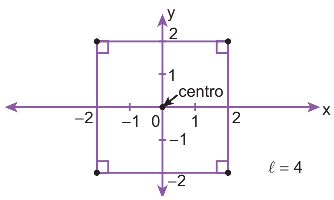
$\therefore$ G(x; y) = G(–1; 2)

Clave B

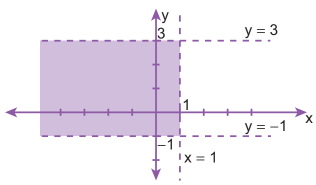
Nivel 2 (página 45) Unidad 3

Comunicación matemática

11. a)



b) Trazamos las rectas que limitarán el área que corresponde a los infinitos puntos.

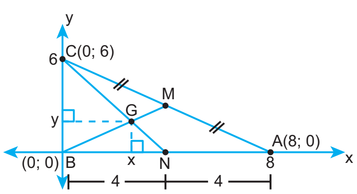


12. Según definición:
Ib – IIc – IIIa

Clave B

Razonamiento y demostración

13.



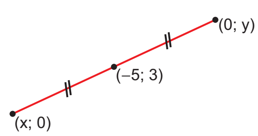
La proyección de $\overline{BG}$ sobre $\overline{AB}$ será el valor de la abscisa del punto G(x; y).
Por dato G es baricentro:

$$\Rightarrow x = \frac{0+0+8}{3} = \frac{8}{3} \qquad \text{(propiedad)}$$

$$\therefore x = \frac{8}{3}$$

Clave C

14.



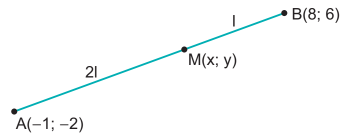
$$-5 = \frac{0+x}{2} \Rightarrow x = -10$$

$$3 = \frac{0+y}{2} \Rightarrow y = 6$$

Piden:
$E = \sqrt{6-(-10)} = \sqrt{16} = 4$
$\therefore E = 4$

Clave C

15.



$$x = \frac{8(2l)+(-1)(l)}{2l+l} = \frac{15l}{3l}$$

$x = 5$

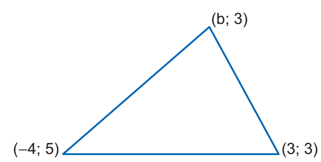
$$y = \frac{6(2l)+(-2)(l)}{2l+l} = \frac{10l}{3l}$$

$$y = \frac{10}{3}$$

$$\therefore M(x;\, y) = M\left(5; \frac{10}{3}\right)$$

Clave C

16.



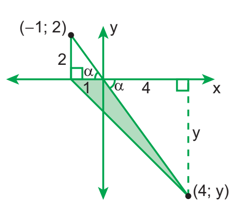
		b	3		
–12	←	–4	5	→	5b
15	←	3	3	→	–12
3b	←	b	3	→	9
(3b + 3)					(5b – 3)
I					D

$$\text{Dato} \rightarrow 10 = \frac{|(5b-3)-(3b+3)|}{2}$$

$\Rightarrow 20 = |2b - 6|$
$\Rightarrow 20 = 2b - 6 \quad \vee \quad 20 = -(2b - 6)$
$\quad 26 = 2b \qquad\qquad 2b = -14$
$\therefore b = 13 \qquad \vee \qquad b = -7$

Clave E

17. Usando distancias:



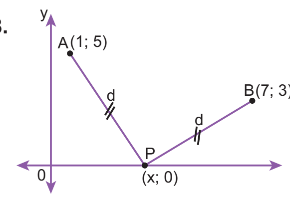
Del gráfico: $\tan\alpha = \frac{2}{1} = \frac{y}{4} \Rightarrow y = 8$

El área del triángulo sombreado es: $\frac{b.h}{2}$
donde: b = 1 $\wedge$ h = y = 8
Reemplazando:

$$\frac{(1)(8)}{2} = 4$$

Clave C

18.



$d^2 = (x-1)^2 + (0-5)^2 \qquad$ ...(I)
$d^2 = (7-x)^2 + (3-0)^2 \qquad$ ...(II)

De (I) y (II):
$(x-1)^2 + (-5)^2 = (7-x)^2 + 3^2$
$x^2 - 2x + 26 = 58 - 14x + x^2$

$$12x = 32 \Rightarrow x = \frac{32}{12}$$

$$x = \frac{8}{3} \Rightarrow \text{El punto es } \left(\frac{8}{3}; 0\right)$$

Clave B

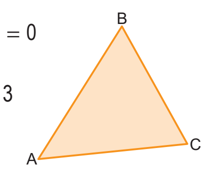
Resolución de problemas

19. Hallamos el vértice C(m; n) mediante la fórmula del baricentro.

$$0 = \frac{-4+4+m}{3} \Rightarrow m = 0$$

$$2 = \frac{1+2+n}{3} \Rightarrow n = 3$$

$\therefore$ C(0; 3)



Para el área tenemos:

		–4	1		
4	←	4	2	→	–8
0	←	0	3	→	12
–12	←	–4	1	→	0
M = –8					4 = N

$$\Rightarrow S_\Delta = \frac{|N-M|}{2}$$

$$S_\Delta = \frac{|4-(-8)|}{2} = \frac{12}{2} = 6$$

$\therefore S_\Delta = 6$

Clave A

20. Reemplazamos en la fórmula del baricentro G(x; y)

$$x = \frac{-3+5+(-8)}{3} = \frac{-6}{3} \Rightarrow x = -2$$

$$y = \frac{-1+4+6}{3} = \frac{9}{3} \Rightarrow y = 3$$

Sumamos:
x + y = –2 + 3 = 1
$\therefore$ x + y = 1

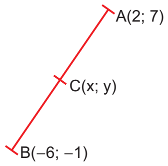
Clave E

Nivel 3 (página 46) Unidad 3

Comunicación matemática

21.

- P:



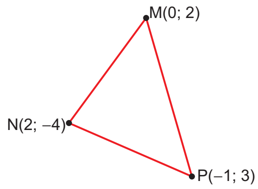
$x = \dfrac{2+(-6)}{2} = \dfrac{-4}{2} \Rightarrow x = -2$

$y = \dfrac{7+(-1)}{2} = \dfrac{6}{2} \Rightarrow y = 3$

$x + y = -2 + 3 = 1$

$\therefore P = 1$

- Q:



Tomamos los puntos en sentido antihorario.

$$\begin{array}{r|cc|l} & 0 & 2 & \\ 4 \leftarrow & 2 & -4 & \rightarrow 0 \\ 4 \leftarrow & -1 & 3 & \rightarrow 6 \\ 0 \leftarrow & 0 & 2 & \rightarrow -2 \\ \hline 8 & & & 4 \end{array}$$

$S_{\Delta} = \dfrac{|4-8|}{2} = 2 \Rightarrow Q = 2$

$\therefore 2P = Q$

Clave D

22. Por definición:
Ib – IIa – IIIc

Clave E

Razonamiento y demostración

23. Los vértices del triángulo son:

A(–1; –3), B(4; 5), C(6; –8)

Sea G(x; y) el baricentro del triángulo:

$\Rightarrow x = \dfrac{(-1)+4+6}{3} = \dfrac{9}{3} = 3$

$y = \dfrac{(-3)+5+(-8)}{3} = \dfrac{-6}{3} = -2$

$\Rightarrow G(x, y) = G(3; -2)$

Piden: $x + y = 3 + (-2) = 3 - 2 = 1$

Clave E

24. Un punto en el eje de ordenadas tiene la forma: (0; y)

$\Rightarrow 17^2 = [0-(-8)]^2 + (y-13)^2$

$17^2 = 8^2 + (y-13)^2$

$289 = 64 + (y-13)^2$

$225 = (y-13)^2$

$\pm 15 = (y-13)$

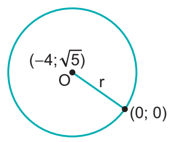
$y - 13 = 15 \quad \vee \quad y - 13 = -15$

$y = 28 \quad \vee \quad y = -2$

$\therefore$ El punto puede ser: (0; –2) $\vee$ (0; 28)

Clave B

25.



$r^2 = (-4-0)^2 + (\sqrt{5}-0)^2$

$r^2 = (-4)^2 + (\sqrt{5})^2$

$r^2 = 16 + 5 = 21$

$\Rightarrow r^2 = 21$

Piden el área:

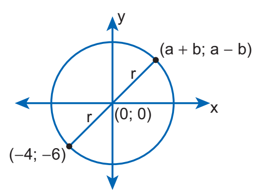
$S_{\odot} = \pi r^2$

$S_{\odot} = \left(\dfrac{22}{7}\right)(21) = 66$

$\therefore S_{\odot} = 66$

Clave D

26.



$r^2 = (-4-0)^2 + (-6-0)^2$

$r^2 = (-4)^2 + (-6)^2$

$r^2 = 16 + 36 = 52$

$r^2 = 52$

También:

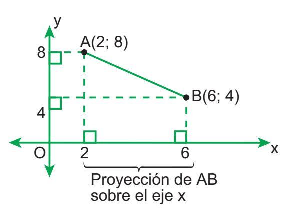
$r^2 = [(a+b)-0]^2 + [(a-b)-0]^2$

$\underbrace{r^2}_{52} = \underbrace{(a+b)^2 + (a-b)^2}_{2(a^2+b^2)}$

$\therefore a^2 + b^2 = 26$

Clave A

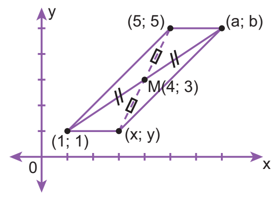
27.



Entonces, proyección de $\overline{AB}$ sobre el eje x será:
$6 - 2 = 4$

Clave A

28.



M es punto medio:

$\left.\begin{array}{l} \Rightarrow 4 = \dfrac{1+a}{2} \Rightarrow a = 7 \\ \Rightarrow 3 = \dfrac{1+b}{2} \Rightarrow b = 5 \end{array}\right\} (a; b) = (7; 5)$

También:

$\left.\begin{array}{l} 4 = \dfrac{5+x}{2} \Rightarrow x = 3 \\ 3 = \dfrac{5+y}{2} \Rightarrow y = 1 \end{array}\right\} (x; y) = (3; 1)$

Clave E

Resolución de problemas

29. Hallamos el lado del cuadrado.

$l = \sqrt{(3-1)^2 + (7-1)^2}$

$l = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$

El área OCD es la cuarta parte del área del cuadrado ABCD.

$A_{\square} = l^2 \Rightarrow A_{\square} = (2\sqrt{10})^2$

$A_{\square} = 40$

$\therefore A_{OCD} = A/4 = 10$

Clave A

30. Hallamos el baricentro G(x; y).

$x = \dfrac{x_1 + x_2 + x_3}{3}$

$x = \dfrac{-6+2+4}{3} = \dfrac{0}{3} = 0$

$y = \dfrac{y_1 + y_2 + y_3}{3}$

$y = \dfrac{-2-3+5}{3} = \dfrac{0}{3} = 0 \quad \therefore G(x; y) = G(0; 0)$

La distancia entre G y A:

$d = \sqrt{(0-(-6))^2 + (0-(-2))^2}$

$d = \sqrt{6^2 + 2^2} = \sqrt{40}$

$\therefore d = 2\sqrt{10}$

Clave C

RAZONES TRIGONOMÉTRICAS DE UN ÁNGULO EN CUALQUIER MAGNITUD

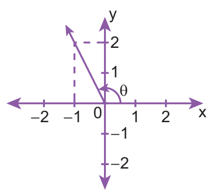
APLICAMOS LO APRENDIDO (página 48) Unidad 3

1.
- $\alpha \in IIC \Rightarrow sen\alpha > 0$ (V)
- $\alpha \in IVC \Rightarrow tan\alpha < 0$ (V)
- $\alpha \in IC \Rightarrow sec\alpha < 0$ (F)
- $\alpha \in IIIC \Rightarrow cos\alpha > 0$ (F)

$\therefore$ VVFF

Clave D

2.



$x = -1$
$y = 2$

$x^2 + y^2 = r^2 \Rightarrow (-1)^2 + (2)^2 = r^2$

$5 = r^2 \Rightarrow r = \sqrt{5}$

Reemplazamos en:

$J = (sen\theta - cos\theta)^2 = \left(\frac{2}{\sqrt{5}} - \left(-\frac{1}{\sqrt{5}}\right)\right)^2$

$J = \left(\frac{3}{\sqrt{5}}\right)^2 = \frac{9}{5}$

Clave B

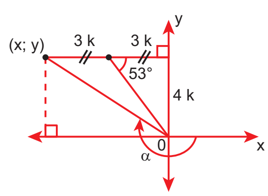
3. Reemplazamos en F(x):

$F(x) = F(180°) = \dfrac{\cos 90° + \cos 360° + \cos 270°}{\sec 360° - \cos 180°}$

$= \dfrac{0 + 1 + 0}{1 - (-1)} = \dfrac{1}{2}$

Clave A

4.



$\Rightarrow (x; y) = (-6k; 4k)$

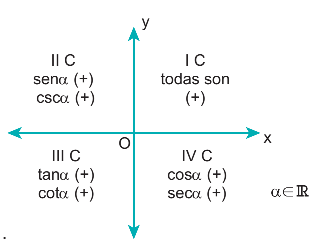
$tan\alpha = \dfrac{y}{x} = \dfrac{4k}{-6k} = \dfrac{-2}{3}$

Piden:

$E = 3tan\alpha + 1 = 3\left(\frac{-2}{3}\right) + 1$
$E = -2 + 1 = -1$

Clave C

5.



$\Rightarrow$ IC: cos y tan son positivas.
$\Rightarrow$ IIC: cos y tan son negativas.
$\therefore$ IC y IIC tienen el mismo signo.

Clave A

6. Sabemos:

$cos\alpha = -\dfrac{3}{5} = \dfrac{x}{r} \Rightarrow x = -3 \wedge r = 5$

$x^2 + y^2 = r^2 \Rightarrow (-3)^2 + y^2 = 5^2$

$y^2 = 25 - 9 = 16$

$y = 4 \quad \alpha \in IIC$

Hallamos R:

$R = \sqrt{\dfrac{3sen^2\alpha - 4\cos^2\alpha}{-5\tan\alpha}} = \sqrt{\dfrac{3\left(\frac{4}{5}\right)^2 - 4\left(-\frac{3}{5}\right)^2}{-5\left(\frac{4}{-3}\right)}}$

$R = \sqrt{\dfrac{\left(\frac{48}{25}\right) - \left(\frac{36}{25}\right)}{\frac{20}{3}}} = \sqrt{\dfrac{\frac{12}{25}}{\frac{20}{3}}}$

$R = \sqrt{\dfrac{9}{125}}$

$R = \dfrac{3\sqrt{5}}{25}$

Clave C

7. En E: $\cos 124° < 0 \Rightarrow (-)$
$\csc 312° < 0 \Rightarrow (-)$
$sen 115° > 0 \Rightarrow (+)$
$\tan 220° > 0 \Rightarrow (+)$

$E = \dfrac{(-)(-)}{(+)(+)} = \dfrac{(+)}{(+)} = (+)$

$\therefore E = (+)$

En T: $sen 336° < 0 \Rightarrow (-)$
$\tan 218° > 0 \Rightarrow (+)$
$\cos 168° < 0 \Rightarrow (-)$

$T = (-)(+)(-) = (+)$
$\therefore T = (+)$

Clave B

8. Sean α y β los ángulos ($\alpha > \beta$).

$\dfrac{\alpha}{\beta} = \dfrac{6}{1} \Rightarrow \begin{matrix} \alpha = 6k \\ \beta = k \end{matrix}$

Sabemos:
$\alpha - \beta = 360°n$
$6k - k = 360°n$
$5k = 360°n$
$k = 72°n$

$800° < 6k + k < 1060°$
$800° < 7k < 1000°$
$114{,}28 < k < 150°$
$114{,}28 < 72n < 150°$
$1{,}58 < n < 2{,}08 \Rightarrow n = 2$
$k = 144$

$\therefore \alpha = 6k = 6(144°)$
$\alpha = 864°$

Clave E

9. De (α):
$(x; y) = (-2; -3) \Rightarrow x = -2 \wedge y = -3$

De (θ):
$(m; n) = (2; -2) \Rightarrow m = 2 \wedge n = -2$

$m^2 + n^2 = r^2 \Rightarrow (2)^2 + (-2)^2 = r^2 \Rightarrow r = 2\sqrt{2}$

Hallamos el valor de R:
$R = cot\alpha + sen\theta - tan\alpha \,.\, tan\theta$

$R = \left(\frac{x}{y}\right) + \left(\frac{n}{r}\right) - \left(\frac{y}{x}\right)\left(\frac{n}{m}\right)$

$R = \left(\frac{-2}{-3}\right) + \left(\frac{-2}{2\sqrt{2}}\right) - \left(\frac{-3}{-2}\right)\left(\frac{-2}{2}\right)$

$R = \frac{2}{3} - \frac{\sqrt{2}}{2} + \frac{3}{2}$

$\therefore R = \dfrac{(13 - 3\sqrt{2})}{6}$

Clave B

10. $f(\theta) = |\cos 3\theta| + \sqrt{1 - sen^2 2\theta} - \cos 2\theta$

$f\left(-\frac{\pi}{3}\right) \Rightarrow \theta = -\frac{\pi}{3} = -60°$

Entonces:

$f\left(-\frac{\pi}{3}\right) = |\cos(-180°)| + \sqrt{1 - sen^2(-120°)} - \cos(-120°)$

$f\left(-\frac{\pi}{3}\right) = |\cos 180°| + \sqrt{1 - sen^2 120°} - \cos 120°$

$f\left(-\frac{\pi}{3}\right) = |-1| + \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2} - \left(-\frac{1}{2}\right)$

$f\left(-\frac{\pi}{3}\right) = 1 + \sqrt{\frac{1}{4}} + \frac{1}{2} = 1 + \frac{1}{2} + \frac{1}{2} = 2$

$f\left(\frac{\pi}{3}\right) \Rightarrow \theta = \frac{\pi}{3} = 60°$

$f\left(\frac{\pi}{3}\right) = |\cos(180°)| + \sqrt{1 - sen^2(120°)} - \cos(120°)$

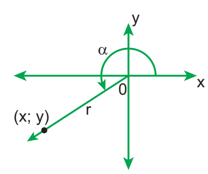
$f\left(\frac{\pi}{3}\right) = |-1| + \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2} - \left(-\frac{1}{2}\right)$

$f\left(\frac{\pi}{3}\right) = 1 + \sqrt{\frac{1}{4}} + \frac{1}{2} = 1 + \frac{1}{2} + \frac{1}{2} = 2$

$\therefore \underbrace{f\left(-\frac{\pi}{3}\right)}_{2} + \underbrace{f\left(\frac{\pi}{3}\right)}_{2} + 1 = 5$

Clave C

11.



$\tan\alpha = \dfrac{1}{3} = \dfrac{-1}{-3} = \dfrac{y}{x}$

$y = -1 \wedge x = -3$

$r^2 = x^2 + y^2$

$r^2 = (-3)^2 + (-1)^2 = 9 + 1 = 10$

$r = \sqrt{10}$

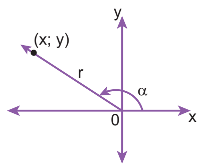
Piden: $P = 3sec\alpha - csc\alpha$

$P = 3\left(\frac{r}{x}\right) - \left(\frac{r}{y}\right) = 3\left(\frac{\sqrt{10}}{-3}\right) - \left(\frac{\sqrt{10}}{-1}\right)$

$= -\sqrt{10} + \sqrt{10} = 0$

Clave B

12. $\cos\alpha = -\left(\dfrac{p^2 - q^2}{p^2 + q^2}\right)$; $p > q > 0$



$\cos\alpha = \dfrac{-(p^2 - q^2)}{p^2 + q^2} = \dfrac{x}{r}$

$\Rightarrow x = -(p^2 - q^2) \ \wedge \ r = p^2 + q^2$

Sabemos: $r^2 = x^2 + y^2$

$(p^2 + q^2)^2 = [-(p^2 - q^2)]^2 + y^2$

$(p^2 + q^2)^2 = (p^2 - q^2)^2 + y^2$

$y^2 = \underbrace{(p^2 + q^2)^2 - (p^2 - q^2)^2}$

$y^2 = 4p^2q^2$ (como: $p > q > 0$)

$\Rightarrow y = 2pq$

Piden: $\tan\alpha = \dfrac{y}{x} = \dfrac{2pq}{-(p^2 - q^2)} = \dfrac{2pq}{q^2 - p^2}$

Clave B

13. Como son coterminales sus funciones son las mismas.

$\text{sen}\alpha = \text{sen}\beta$

$\cos\alpha = \cos\beta$

En k, tenemos:

$k = \dfrac{(1 - \text{sen}^2\alpha)(1 - \cos^2\alpha)}{[-(1 - \cos^2\alpha)][-(1 - \text{sen}^2\alpha)]}$

$k = \dfrac{(1 - \text{sen}^2\alpha)(1 - \cos^2\alpha)}{(1 - \cos^2\alpha)(1 - \text{sen}^2\alpha)}$

$\therefore \ k = 1$

Clave C

14. $\theta \in \langle 40°; 100°]$

$40° < \theta \leq 100°$

$20° < \dfrac{\theta}{2} \leq 50°$

$\dfrac{\theta}{2} \in \text{IC}$

$40° < \theta \leq 100°$

$10° < \dfrac{\theta}{4} \leq 25°$

$\dfrac{\theta}{4} \in \text{IC}$

$P = \tan\dfrac{\theta}{2} + \cos\left(-\dfrac{\theta}{4}\right)$

$P = \tan\dfrac{\theta}{2} + \cos\left(\dfrac{\theta}{4}\right)$

$P = (+) + (+) = (+)$

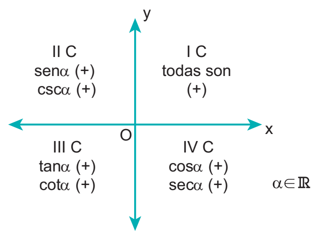
Clave A

PRACTIQUEMOS

Nivel 1 (página 50) Unidad 3

Comunicación matemática

1. Reconocemos el signo de las razones en cada cuadrante:



Entonces:

– Si $\alpha \in$ IIC $\Rightarrow$ cosα, es (–)

– Si $\alpha \in$ IIIC $\Rightarrow$ tanα, es (+)

– Si $\alpha \in$ IC $\Rightarrow$ senα, es (+)

– Si $\alpha \in$ IVC $\Rightarrow$ cotα, es (–)

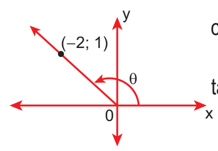
– Si $\alpha \in$ IIC $\Rightarrow$ senα . cosα, es (–)

– Si $\alpha \in$ IVC $\Rightarrow$ tanα . senα, es (+)

2.

Razonamiento y demostración

3.



$\cot\theta = \dfrac{x}{y} = \dfrac{-2}{1} = -2$

$\tan\theta = \dfrac{y}{x} = \dfrac{1}{-2} = -\dfrac{1}{2}$

Piden: $S = \tan\theta - \cot\theta$

$-\dfrac{1}{2} - (-2) = -\dfrac{1}{2} + 2 = \dfrac{3}{2}$

Clave A

4. $P = \dfrac{\text{sen}200° - \cos 310°}{\tan 140°} = \dfrac{(-) - (+)}{(-)} = \dfrac{(-)}{(-)}$

$P = \dfrac{(-)}{(-)} = (+)$

Clave A

5. $\underbrace{\tan\beta < 0} \ \wedge \ \underbrace{\text{sen}\beta < 0}$

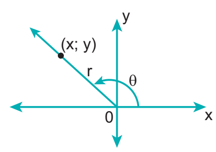
$\beta \in \text{IIC} \vee \text{IVC} \ \wedge \ \beta \in \text{IIIC} \vee \text{IVC}$

De ambas condiciones:

$\beta \in \text{IVC}$

Clave D

6. $\cos\theta = -\dfrac{2}{3}; \tan\theta < 0 \Rightarrow \underbrace{\csc\theta < 0 \wedge \tan\theta < 0}_{\theta \in \text{IIC}}$



$\cos\theta = -\dfrac{2}{3} = \dfrac{x}{r}$

$x = -2, \ r = 3 \Rightarrow y = \sqrt{5}$

$\tan\theta = \dfrac{y}{x} = \dfrac{\sqrt{5}}{-2} = -\dfrac{\sqrt{5}}{2}$

$\text{sen}\theta = \dfrac{y}{r} = \dfrac{\sqrt{5}}{3}$

Piden: $T = \sqrt{5}\tan\theta + \dfrac{1}{\sqrt{5}}\text{sen}\theta$

$T = \sqrt{5}\left(-\dfrac{\sqrt{5}}{2}\right) + \dfrac{1}{\sqrt{5}}\left(\dfrac{\sqrt{5}}{3}\right)$

$T = -\dfrac{5}{2} + \dfrac{1}{3} = \dfrac{-15 + 2}{6}$

$\therefore T = -\dfrac{13}{6}$

Clave C

7. $\underbrace{\underbrace{\tan\theta}_{(-)}\underbrace{\sqrt[3]{\text{sen}\theta}}_{(-)} > 0; \underbrace{\cos\theta}_{(+)}\underbrace{\sqrt[3]{\text{sen}\theta}}_{(-)} < 0}$

cumple $\forall \ \theta \in \text{IVC}$

Además: $\underbrace{|\tan\theta|}_{(-)} = 3 \quad \theta \in \text{IVC}$

$\Rightarrow -(\tan\theta) = 3$

$\tan\theta = \underbrace{-3}$

$\dfrac{y}{x} = \dfrac{-3}{1}$

$y = -3, x = 1 \Rightarrow r = \sqrt{10}$

Piden:

$K = \text{sen}\theta + \cos\theta$

$K = \dfrac{y}{r} + \dfrac{x}{r} = \dfrac{-3}{\sqrt{10}} + \dfrac{1}{\sqrt{10}} = -\dfrac{2}{\sqrt{10}} = -\sqrt{\dfrac{4}{10}}$

$\therefore K = -\sqrt{0{,}4}$

Clave D

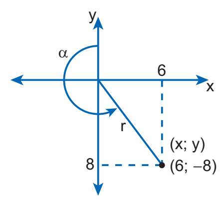
8. $\sec\theta < 0 \ \wedge \ \cot\theta > 0$

$\Rightarrow \theta \in (\text{IIC} \ \vee \ \text{IIIC}) \ \wedge \ \theta \in (\text{IC} \vee \text{IIIC})$

$\therefore$ De ambas condiciones $\theta \in \text{IIIC}$

Clave C

9.



$r^2 = 6^2 + 8^2 \Rightarrow r = 10$

$E = 5\cos\alpha + 6\tan\alpha$

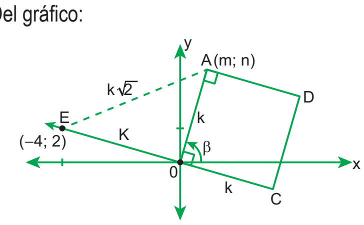
$E = 5\left(\dfrac{6}{10}\right) + 6\left(-\dfrac{8}{6}\right)$

$E = 3 - 8 = -5$

Clave C

Resolución de problemas

10. Del gráfico:



$k = \sqrt{(-4-0)^2 + (2-0)^2}$

$k^2 = 16 + 4 = 20 \Rightarrow k^2 = 20$

Hallamos el segmento AO:

$AO = k = \sqrt{(m-0)^2 + (n-0)^2}$

$k^2 = m^2 + n^2 \Rightarrow 20 = m^2 + n^2$

Hallamos el segmento AE:

$AE = k\sqrt{2} = \sqrt{(m-(-4))^2 + (n-2)^2}$

$2k^2 = (m+4)^2 + (n-2)^2$

$2(20) = m^2 + 8m + 16 + n^2 - 4n + 4$

$40 = m^2 + n^2 + 20 + 8m - 4n$

$40 = 20 + 20 + 8m - 4n$

$0 = 8m - 4n \Rightarrow 2m = n$

Reemplazamos:

$20 = (2m)^2 + m^2 = 5m^2$

$4 = m^2 \Rightarrow m = 2 \wedge n = 4$

Hallamos M:

$M = \tan\beta + 1$

$M = \frac{4}{2} + 1 = 2 + 1 \qquad \therefore M = 3$

Clave C

11. $\beta = (1°)^2 + (2°)^2 + (3°)^2 + ... + (n°)^2, n \in \mathbb{Z}$

$\beta = \frac{n(n+1)(2n+1)}{6}$

$\beta°_{máx} < 720°$

$\frac{n(n+1)(2n+1)}{6} < 720°$

$n°(n°+1)(2n°+1) < 12 \times 12 \times 30°$

Con $\beta_{máx.} \Rightarrow n_{máx.}$

$\therefore n_{máx.} = 12$

Clave D

Nivel 2 (página 50) Unidad 3

Comunicación matemática

12. Definimos el cuadrante al que pertenece cada ángulo.

$\beta \in IIC; \theta \in IVC; \gamma \in IIIC$

$\operatorname{sen}\beta \quad \cos\theta \quad \tan\gamma$

$\csc\beta \quad \sec\theta \quad \cot\gamma$

13. I. Si $\theta \in IVC$

$\Rightarrow \sec\theta > 0$

$\tan\theta < 0$

$\sec\theta \,.\, \tan\theta < 0$

$k < 0$

I. Falso

II. Si $\theta \in IIIC$

$\Rightarrow \sec\theta < 0$

$\tan\theta > 0$

$\sec\theta \,.\, \tan\theta < 0 \Rightarrow k < 0$

$\Rightarrow |k| = -\sec\theta \,.\, \tan\theta$

II. Verdadero

III. Si $\theta \in IIC$

$\Rightarrow \sec\theta < 0$

$\tan\theta < 0$

$\sec\theta \,.\, \tan\theta > 0 \Rightarrow k > 0$

$\Rightarrow |k| = \sec\theta \,.\, \cos\theta$

III. Falso

IV. Si $k > 0$

$\Rightarrow \sec\theta \,.\, \tan\theta > 0$

$(+) \wedge (+)$

$(-) \wedge (-)$

Si $\sec\theta > 0 \wedge \tan\theta > 0 \Rightarrow \theta \in IC$

Si $\sec\theta < 0 \wedge \tan\theta < 0 \Rightarrow \theta \in IIC$

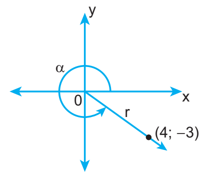
IV. Verdadero

$\Rightarrow$ Solo II y IV.

Clave E

Razonamiento y demostración

14.



$r^2 = 4^2 + (-3)^2 = 25$

$r = 5$

$\operatorname{sen}\alpha = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$

$\cos\alpha = \frac{x}{r} = \frac{4}{5}$

Piden: $S = \underbrace{\operatorname{sen}\alpha}_{\frac{-3}{5}} + \underbrace{\cos\alpha}_{\frac{4}{5}} = \frac{1}{5} = 0,2$

Clave A

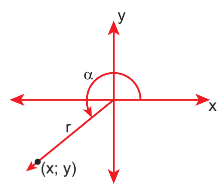
15. $\underbrace{\operatorname{sen}\theta < 0}_{\theta \in IIIC \vee IVC} \wedge \underbrace{\cos\theta < 0}_{\theta \in IIC \vee IIIC}$

De ambas condiciones:

$\theta \in IIIC$

Clave C

16. $\underbrace{\cot\alpha}_{>0} = \frac{1}{3}; \underbrace{|\operatorname{sen}\alpha|}_{<0} = -\operatorname{sen}\alpha \quad \Rightarrow \alpha \in IIIC$



$\cot\alpha = \frac{1}{3} = \frac{-1}{-3} = \frac{x}{y}$

$x = -1, y = -3 \Rightarrow r = \sqrt{10}$

$\operatorname{sen}\alpha = \frac{y}{r} = \frac{-3}{\sqrt{10}} = -\frac{3}{\sqrt{10}}$

$\cos\alpha = \frac{x}{r} = \frac{-1}{\sqrt{10}} = -\frac{1}{\sqrt{10}}$

Piden: $P = \operatorname{sen}\alpha - \cos\alpha$

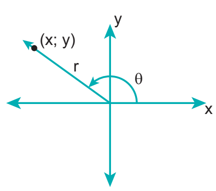
$P = -\frac{3}{\sqrt{10}} - \left(-\frac{1}{\sqrt{10}}\right) = \frac{-3+1}{\sqrt{10}}$

$= -\frac{2}{\sqrt{10}} = -\sqrt{\frac{4}{10}} \quad \therefore P = -\sqrt{\frac{2}{5}}$

Clave E

17. $\cot\theta = \frac{\operatorname{sen}\theta + x}{\cos\theta + x}; \sec\theta = -2,6$

$\theta \in IIC$



$\sec\theta = -2,6 = -\frac{13}{5} = \frac{13}{-5} = \frac{r}{x}$

Por lo tanto:

$r = 13, \quad x = -5 \Rightarrow y = 12$

$\cot\theta = \frac{x}{y} = \frac{-5}{12} = -\frac{5}{12}$

$\operatorname{sen}\theta = \frac{y}{r} = \frac{12}{13}$

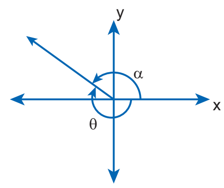
$\cos\theta = \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13}$

$\Rightarrow -\frac{5}{12} = \frac{\frac{12}{13} + x}{\frac{-5}{13} + x} \Rightarrow \frac{25}{156} - \frac{5x}{12} = \frac{12}{13} + x$

$-\frac{119}{156} = \frac{17}{12}x \quad \therefore x = -\frac{7}{13}$

Clave C

18.



α y θ son ángulos coterminales, entonces:

$RT(\alpha) = RT(\beta)$

Piden:

$L = \frac{3\operatorname{sen}\alpha + \operatorname{sen}\alpha}{3\operatorname{sen}\alpha - \operatorname{sen}\alpha} \Rightarrow L = \frac{4\operatorname{sen}\alpha}{2\operatorname{sen}\alpha} \Rightarrow L = 2$

Clave E

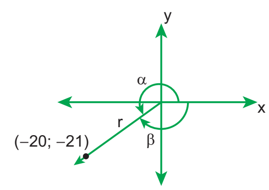
19. $\operatorname{sen}\beta < 0 \quad \wedge \tan\beta < 0$

$\Rightarrow (\beta \in IIIC \vee IVC) \wedge (\beta \in IIC \vee IVC)$

$\Rightarrow$De ambas condiciones $\beta \in IVC$.

Clave D

20.



$\operatorname{sen}\alpha = \frac{-21}{r}$

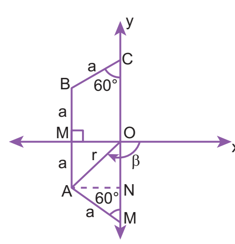
$\csc\beta = \frac{r}{-21}$

$\Rightarrow E = \operatorname{sen}\alpha \,.\, \csc\beta \qquad E = \frac{-21}{r} \cdot \frac{r}{-21} = 1$

Clave A

Resolución de problemas

21.



El punto A:

$A(x; y) = \left(-a\frac{\sqrt{3}}{2}; -a\right)$

$r^2 = x^2 + y^2 = \left(-a\frac{\sqrt{3}}{2}\right)^2 + (-a)^2$

$r^2 = \frac{7}{4}a^2 \Rightarrow r = \sqrt{\frac{7}{4}}\,a$

$\text{sen}\beta = \frac{y}{r} = \frac{-a}{\sqrt{\frac{7}{4}}\,a} = \frac{-2\sqrt{7}}{7}$

Clave A

22. Sean los ángulos α y β, $\alpha > \beta$

$\frac{\alpha}{\beta} = \frac{3}{2} \Rightarrow \begin{array}{l}\alpha = 3k\\ \beta = 2k\end{array}$

Por ángulos coterminales se cumple:

$\alpha - \beta = 360n,\ n \in \mathbb{Z}$

$3k - 2k = 360n$

$k = 360n$

Si $n = 1 \Rightarrow k = 36° \Rightarrow \left.\begin{array}{l}\alpha = 1080°\\ \beta = 720°\end{array}\right\}\checkmark$

Si $m = 2 \Rightarrow k = 720° \Rightarrow \left.\begin{array}{l}\alpha = 2160°\\ \beta = 1440°\end{array}\right\}\checkmark$

Si $n = 3 \Rightarrow k = 1080° \Rightarrow \left.\begin{array}{l}\alpha = 3240°\\ \beta = 2160°\end{array}\right\}\times$

$\Sigma\text{valores} = 720° + 1440° = 2160°$

Clave E

Nivel 3 (página 51) Unidad 3

Comunicación matemática

23. $M = 2^{\text{sen}270°} - 3^{\cot 90°} + \frac{2\cos 0° + \text{sen}270°}{\tan 180° + \cos 360°}$

$M = 2^{-1} - 3^0 + \frac{2(1) + (-1)}{0 + 1}$

$M = \frac{1}{2} - 1 + \frac{1}{1} = \frac{1}{2}$

$N = 4^{\sec 0° + \sec 180°} + \frac{3\tan 0° - 2\csc 270°}{\cos 90° - \text{sen}90°}$

$N = 4^{1+(-1)} + \frac{3(0) - 2(-1)}{0 - 1}$

$N = 4^0 + \frac{0 + 2}{-1} \Rightarrow N = -1$

$P = \sqrt{3^{\sec 360° + \csc 90°} - \cot 90° + \cos 360° + \sec 180°}$

$P = \sqrt{3^{1+1} - 0 + 1 + (-1)}$

$P = \sqrt{3^2} \Rightarrow P = 3$

$\therefore\ 6M = -3N = P$

$P + N = 4M$

Clave B

24. En I:

$\left.\begin{array}{l}\alpha \in IC \Rightarrow \sec\alpha > 0\\ \beta \in IIC \Rightarrow \text{sen}\beta > 0\end{array}\right\} A > 0$ (V)

En II:

$\left.\begin{array}{l}\alpha \in IVC \Rightarrow \sec\alpha > 0\\ \beta \in IIIC \Rightarrow \text{sen}\beta < 0\end{array}\right\} A < 0$ (V)

En III:

$\left.\begin{array}{l}\alpha \in IIC \Rightarrow \sec\alpha < 0\\ \beta \in IC \Rightarrow \text{sen}\beta > 0\end{array}\right\} A > 0$ (F)

En IV:

$\left.\begin{array}{l}\alpha \in IIIC \Rightarrow \sec\alpha < 0\\ \beta \in IVC \Rightarrow \text{sen}\beta < 0\end{array}\right\} A > 0$ (F)

$\therefore$ VVFF

Clave C

Razonamiento y demostración

25. $\underbrace{\sqrt{-\text{sen}\theta\underbrace{\sqrt{\cos\theta}}_{>0}}}_{\theta \in IC\ \vee\ IVC} \Rightarrow \cos\theta > 0$

$\underbrace{\underset{(-)}{-\text{sen}\theta}\underset{(+)}{\sqrt{\cos\theta}}}_{>0} \Rightarrow \text{sen}\theta < 0$

$\theta \in IIIC\ \vee\ IVC$

De ambas condiciones:

$\theta \in IVC$

Clave D

26. $Q = \frac{\tan 100° + \cos 130°}{\text{sen}160° - \tan 340°} = \frac{(-)+(-)}{(+)-(-)} = \frac{(-)}{(+)}$

$Q = \frac{(-)}{(+)} = (-)$

$R = \frac{\text{sen}100°.\cos 200° - R\cos 190°}{\cos 170°}$

$R\cos170° = \text{sen}100° . \cos200° - R\cos190°$

$R\cos170° + R\cos190° = \text{sen}100° . \cos200°$

$R(\cos170° + \cos190°) = \text{sen}100° . \cos200°$

$R = \frac{\text{sen}100°.\cos 200°}{\cos 170° + \cos 190°} = \frac{(+)(-)}{(-)+(-)} = \frac{(-)}{(-)}$

$\Rightarrow R = (+) \qquad \therefore (-); (+)$

Clave D

27. $\underbrace{\tan\alpha > 0}_{\alpha \in IC \vee IIIC} \wedge \underbrace{\text{sen}\alpha < 0}_{\alpha \in IIIC \vee IVC}$

De ambas condiciones: $\alpha \in IIIC$

$180° < \alpha < 270°$

$\underbrace{90° < \frac{\alpha}{2} < 135°}_{\frac{\alpha}{2} \in IIC} \qquad \underbrace{120° < \frac{2\alpha}{3} < 180°}_{\frac{2\alpha}{3} \in IIC}$

$P = \cos\alpha . \cos\frac{\alpha}{2}$

$P = (-) . (-) = (+)$

$Q = \tan\frac{2\alpha}{3} - \text{sen}\frac{\alpha}{2}$

$Q = (-) - (+) = (-)$

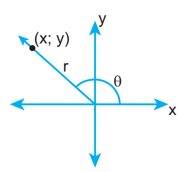
$\therefore (+); (-)$

Clave B

28. $\tan\theta = -\text{sen}45°;\ |\underbrace{\sec\theta}_{(-)}| = -\sec\theta$

$\tan\theta = -\frac{\sqrt{2}}{2}$

$\sec\theta < 0 \Rightarrow 0 \in \theta$ IIC



$\tan\theta = \frac{\sqrt{2}}{-2} = \frac{y}{x}$

$y = \sqrt{2},\quad x = -2 \Rightarrow r = \sqrt{6}$

$\text{sen}\theta = \frac{y}{r} = \frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{3}}{3}$

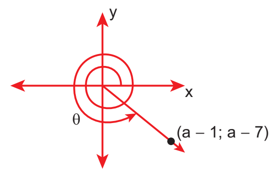
$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{6}} = -\frac{\sqrt{6}}{3}$

Piden: $N = \text{sen}\theta . \cos\theta$

$N = \left(\frac{\sqrt{3}}{3}\right)\left(-\frac{\sqrt{6}}{3}\right) = -\frac{3\sqrt{2}}{3.3} = -\frac{\sqrt{2}}{3}$

Clave D

29.



Si: $\text{sen}\theta + 2\cos\theta = 0$

$\text{sen}\theta = -2\cos\theta$

$\underbrace{\frac{\text{sen}\theta}{\cos\theta}} = -2$

$\tan\theta = -2$

$\tan\theta = -\frac{2}{1} = \frac{2}{-1} = \frac{y}{x} = \frac{a-7}{a-1}$

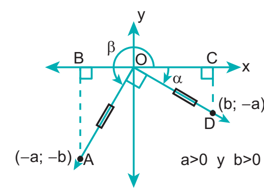
$\Rightarrow \frac{2}{-1} = \frac{a-7}{a-1} \Rightarrow 2a - 2 = 7 - a$

$3a = 9$

$\therefore a = 3$

Clave A

30.



Hacemos $AO = OD \Rightarrow \triangle ABO \cong \triangle OCD$

Si: $A(-a; -b) \Rightarrow D(b; -a)$

Entonces: $\tan\alpha = \frac{y}{x} = \frac{-a}{b} = -\frac{a}{b}$

$\tan\beta = \frac{y}{x} = \frac{-b}{-a} = \frac{b}{a}$

Piden: $\tan\alpha . \tan\beta = \left(-\frac{a}{b}\right)\left(\frac{b}{a}\right) = -1$

Clave B

31. $0° < x < 360°$

$\text{sen}x = \tan2\pi$

$\text{sen}x = 0$

$\Rightarrow x = 180°$

Piden: $\text{sen}\left(\frac{x}{2}\right) + \cot\left(\frac{x}{4}\right) + \csc\left(\frac{x}{6}\right)$

$= \text{sen}(90°) + \cot(45°) + \csc(30°)$

$= 1 + 1 + 2 = 4$

Clave D

32. $\underbrace{\underbrace{\text{sen}\beta}_{(-)}\underbrace{\sqrt{-\tan\beta}}_{(-)}}_{(+)} < 0$

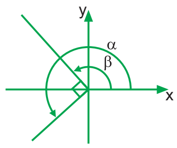
$\text{sen}\beta\ (< 0) \qquad \tan\beta(< 0)$

$(\beta \in \text{IIIC} \vee \beta \in \text{IVC}) \wedge (\beta \in \text{IIC} \vee \beta \in \text{IVC})$
$\therefore \beta \in \text{IVC}$

Clave D

Resolución de problemas

33. $\tan\beta < 0;\ \alpha > \beta$



$\beta \in \text{IIC} \Rightarrow 90° < \beta < 180°$

$45° < \dfrac{\beta}{2} < 90° \Rightarrow \left(\dfrac{\beta}{2}\right) \in \text{IC}$

$180° < 2\beta < 360°$
$(2\beta) \in \text{IIIC} \vee \text{IVC}$
$\alpha \in \text{IIIC} \Rightarrow 180° < \alpha < 270°$

$90° < \dfrac{\alpha}{2} < 135°$

$\Rightarrow \left(\dfrac{\alpha}{2}\right) \in \text{IIC}$

$360° < 2\alpha < 540°$
$(2\alpha) \in \text{IC} \vee \text{IIC}$

Piden los signos de:

$A = \underbrace{\text{sen}\alpha}_{(-)} + \underbrace{\cos\alpha}_{(-)} = (-)$

$N = \underbrace{\cos\dfrac{\alpha}{2}}_{(-)} - \underbrace{\text{sen}\dfrac{\beta}{2}}_{(+)} = (-)$

$F = \underbrace{\text{sen}2\alpha}_{(+)} - \underbrace{\text{sen}2\beta}_{(-)} = (+)$

$\therefore (-); (-); (+)$

Clave A

34. Si $\theta \in \text{IIIC}$:
$\Rightarrow\ 180° < \theta < 270°$

$\dfrac{180°}{3} < \dfrac{\theta}{3} < \dfrac{270°}{3} \Rightarrow 60 < \dfrac{\theta}{3} < 90 \quad ...(\text{I})$

$\Rightarrow \tan\left(\dfrac{\theta}{3}\right)$ es (+)

$\dfrac{180°}{4} < \dfrac{\theta}{4} < \dfrac{270°}{4} \Rightarrow 45° < \dfrac{\theta}{4} < 67{,}5° \quad ...(\text{II})$

$\Rightarrow \text{sen}\left(\dfrac{\theta}{4}\right)$ es (+)

$\dfrac{3(180°)}{4} < \dfrac{3\theta}{4} < \dfrac{3(270°)}{4} \Rightarrow 135° < \dfrac{3\theta}{4} < 182{,}5° \ ...(\text{III})$

$\Rightarrow \cos\left(\dfrac{3\theta}{4}\right)$ es (−)

$A = \dfrac{(+)}{(+)(-)} = (-)$

$\dfrac{180°}{5} < \dfrac{\theta}{5} < \dfrac{270°}{5} \Rightarrow 36° < \dfrac{\theta}{5} < 54° \ ...(\text{IV})$

$\Rightarrow \cos\left(\dfrac{\theta}{5}\right)$ es (+)

$\dfrac{180°}{2} < \dfrac{\theta}{2} < \dfrac{270°}{2} \Rightarrow 90° < \dfrac{\theta}{2} < 135° \ ...(\text{V})$

$\Rightarrow \tan\left(\dfrac{\theta}{2}\right)$ es (−)

$B = \dfrac{(+)(+)}{(-)(+)} = (-)$

$C = \dfrac{(-)(+)}{(+)(+)} = (-)$

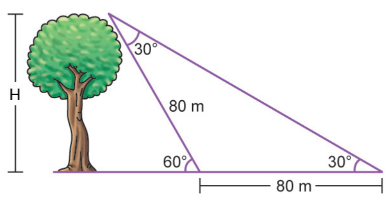
$\therefore\ (-)(-)(-)$

Clave B

ÁNGULOS VERTICALES

APLICAMOS LO APRENDIDO (página 53) Unidad 3

1.



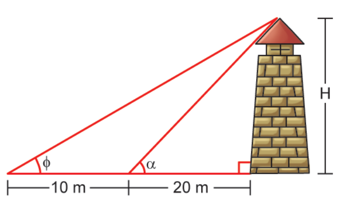
Del gráfico: $\text{sen}60° = \dfrac{H}{80}$

$\dfrac{\sqrt{3}}{2} = \dfrac{H}{80} \Rightarrow H = \dfrac{80\sqrt{3}}{2}$

$\therefore H = 40\sqrt{3}$ m

Clave B

2.



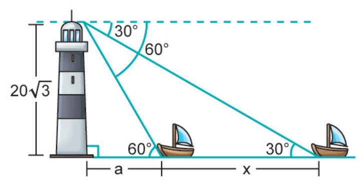
Por dato:
$\underbrace{\tan\alpha}_{} = 3$

$\dfrac{H}{20} = 3 \Rightarrow H = 60$ m

Piden: $\tan\phi = \dfrac{H}{30} = \dfrac{60}{30} = 2 \quad \therefore \tan\phi = 2$

Clave D

3.

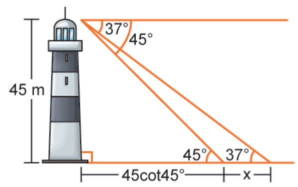


Del gráfico:
$a = 20$
Además: $a + x = 20\sqrt{3}(\sqrt{3})$
$ a + x = 60$
$ \downarrow$
$ 20$

$\therefore x = 40$

Clave C

4.



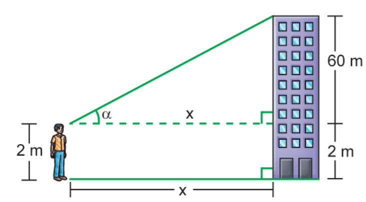
Del gráfico:
$x + 45\cot45° = 45\cot37°$

$x + 45(1) = 45\left(\dfrac{4}{3}\right) \Rightarrow x + 45 = 60$

$\therefore\ x = 15$ m

Clave A

5.



Dato: $\tan\alpha = 0{,}6 = \dfrac{3}{5}$

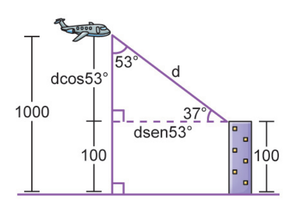
Del gráfico: $\tan\alpha = \dfrac{60}{x}$

Entonces: $\dfrac{3}{5} = \dfrac{60}{x} \Rightarrow x = \dfrac{60.5}{3} = 100$

$\therefore x = 100$ m

Clave D

6.



Piden: la distancia del avión al último piso del edificio (d).
Del gráfico:

$d\cos 53° + 100 = 1000$

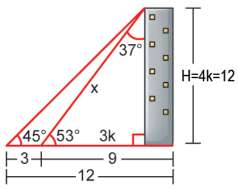
$d\cos 53° = 900$

$d\left(\frac{3}{5}\right) = 900$

$\therefore d = 1500$ m

Clave B

7. Considerar:
Debe avanzar 3 m
La distancia de la primera posición hasta la torre debe ser 12 m.



Por $\triangle$ notable de 37° y 53°:

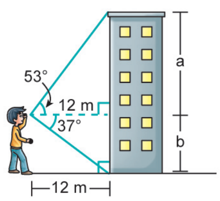
$3k = 9$

$k = 3$

$\Rightarrow x = 5k \qquad \therefore x = 15$ m

Clave C

8.



Por $\triangle$ notable de 37° y 53°:

$a = 16$ m

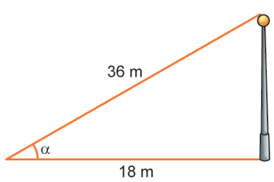
$b = 9$ m

$\Rightarrow$ La altura del edificio: $(a + b) = 16 + 9$

$\therefore (a + b) = 25$ m

Clave A

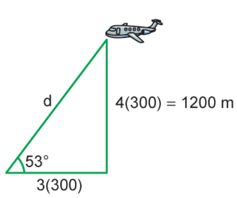
9.



Del gráfico: $\cos\alpha = \frac{18}{36} = \frac{1}{2} \Rightarrow \alpha = 60°$

Clave A

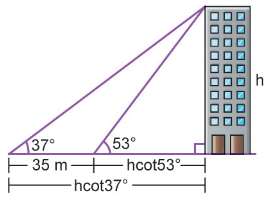
10.



$d = 5(300) = 1500$ m

Clave C

11.



Del gráfico:

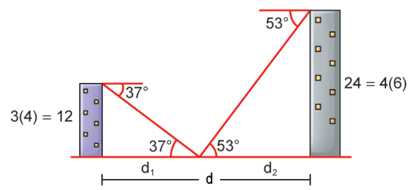
$h\cot 37° = 35 + h\cot 53°$

$\frac{4}{3}h - \frac{3}{4}h = 35$

$\frac{7h}{12} = 35 \Rightarrow h = 60$ m

Clave B

12.



Del gráfico: $d = d_1 + d_2$

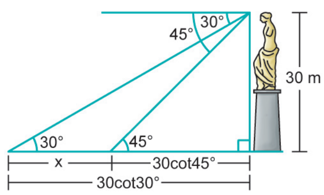
$d = 4(4) + 3(6)$

$d = 16 + 18$

$d = 34$ m

Clave D

13.



Del gráfico: $x + 30\cot 45° = 30\cot 30°$

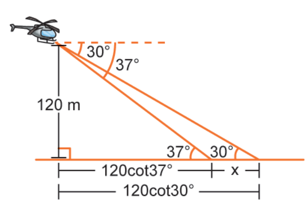
$x + 30(1) = 30(\sqrt{3})$

$x + 30 = 30\sqrt{3}$

$\therefore x = 30(\sqrt{3} - 1)$ m

Clave B

14.



Del gráfico:

$120\cot 30° = 120\cot 37° + x$

$120(\sqrt{3}) = 120\left(\frac{4}{3}\right) + x$

$120\sqrt{3} = 160 + x$

$120\sqrt{3} - 160 = x$

$\therefore x = 40(3\sqrt{3} - 4)$ m

Clave B

PRACTIQUEMOS

Nivel 1 (página 55) Unidad 3

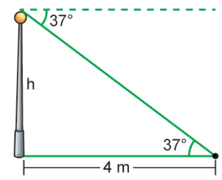
Comunicación matemática

1.

2.

Razonamiento y demostración

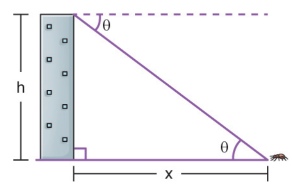
3.



$h = 4\tan 37° \qquad \therefore h = 3$ m

Clave E

4.

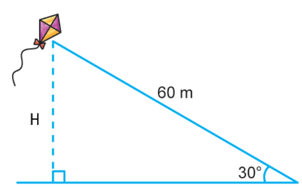


$\therefore x = h\cot\theta$

Clave C

Resolución de problemas

5.



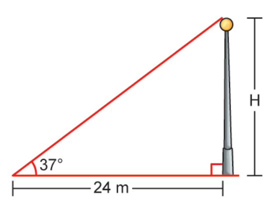
Del gráfico: $\underbrace{\text{sen}30°}_{\frac{1}{2}} = \frac{H}{60}$

$\Rightarrow \frac{1}{2} = \frac{H}{60} \Rightarrow H = \frac{60}{2} = 30$

$\therefore H = 30$ m

Clave D

6.



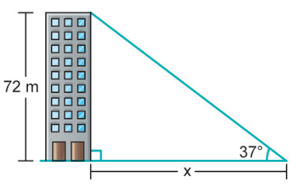
Del gráfico: $\underbrace{\tan 37°}_{\frac{3}{4}} = \frac{H}{24}$

$\frac{3}{4} = \frac{H}{24} \Rightarrow H = \frac{24.3}{4} = 18$

$\therefore H = 18$ m

Clave B

7.



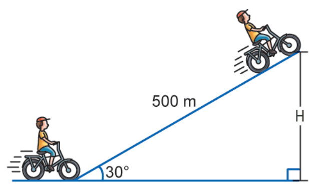
Del gráfico: $\cot 37° = \dfrac{x}{72}$

$$\frac{4}{3} = \frac{x}{72} \Rightarrow x = \frac{72.4}{3} = 96$$

$\therefore x = 96$ m

Clave C

8.



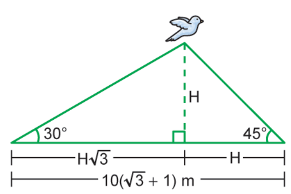
Del gráfico: $\underbrace{\text{sen}30°}= \dfrac{H}{500}$

$$\frac{1}{2} = \frac{H}{500} \Rightarrow H = \frac{500}{2} = 250$$

$\therefore H = 250$ m

Clave A

9.



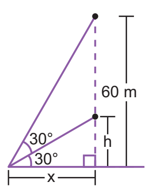
$$H\sqrt{3} + H = 10(\sqrt{3} + 1)$$

$$H(\sqrt{3} + 1) = 10(\sqrt{3} + 1)$$

$\therefore H = 10$ m

Clave A

10.



Del gráfico:

$$x = 60\cot 60° = 60\left(\frac{\sqrt{3}}{3}\right) = 20\sqrt{3}\text{ m}$$

$$h = x\tan 30° = 20\sqrt{3}\left(\frac{1}{\sqrt{3}}\right) = 20$$

$\therefore h = 20$ m

Clave C

Nivel 2 (página 55) Unidad 3

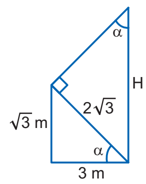
Comunicación matemática

11.

12.

Razonamiento y demostración

13.



Del gráfico:

$\text{sen}\alpha = \dfrac{1}{2}$

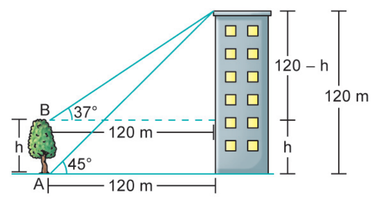
$\text{sen}\alpha = \dfrac{2\sqrt{3}}{H}$

Igualando:

$\dfrac{2\sqrt{3}}{H} = \dfrac{1}{2}$ $\qquad \therefore H = 4\sqrt{3}$ m

Clave C

14.



Del gráfico:

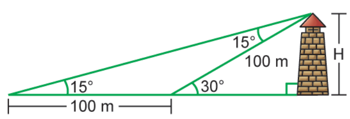
$$\tan 37° = \frac{120 - h}{120}$$

$\dfrac{3}{4} = \dfrac{120 - h}{120}$ $\qquad \therefore h = 30$ m

Clave A

Resolución de problemas

15.



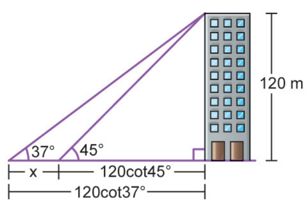
Del gráfico: $\underbrace{\text{sen}30°} = \dfrac{H}{100}$

$$\frac{1}{2} = \frac{H}{100} \Rightarrow H = \frac{100}{2}$$

$\therefore H = 50$ m

Clave B

16.



Del gráfico: $x + 120\cot 45° = 120\cot 37°$

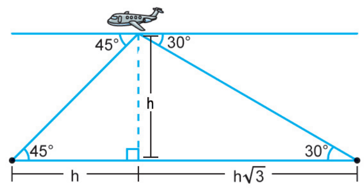
$$x + 120(1) = 120\left(\frac{4}{3}\right)$$

$$x + 120 = 160$$

$$\therefore x = 40\text{ m}$$

Clave A

17.



Por dato: $h = 4(\sqrt{3} + 1)$ km

Del gráfico la distancia de separación entre los puntos es:

$h + h\sqrt{3} = h(\sqrt{3} + 1)$

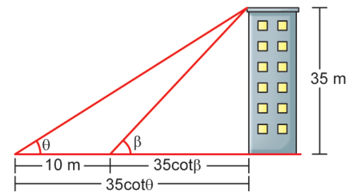
Reemplazando el valor de h:

$$4(\sqrt{3} + 1)(\sqrt{3} + 1) = 4(4 + 2\sqrt{3})$$

$$= (16 + 8\sqrt{3})\text{ km}$$

Clave E

18.



Piden: $E = \cot\theta - \cot\beta$

Del gráfico: $35\cot\theta = 35\cot\beta + 10$

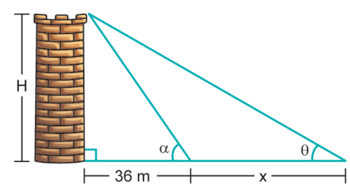
$$35(\cot\theta - \cot\beta) = 10$$

$$\underbrace{\cot\theta - \cot\beta} = \frac{10}{35}$$

$$\therefore E = \frac{2}{7}$$

Clave D

19.



Por dato: $\tan\alpha = \dfrac{7}{12} \quad \wedge \quad \tan\theta = \dfrac{1}{4}$

Del gráfico: $\underbrace{\tan\alpha} = \dfrac{H}{36}$

$$\frac{7}{12} = \frac{H}{36} \Rightarrow H = 21$$

$$\underbrace{\tan\theta} = \frac{H}{36 + x}$$

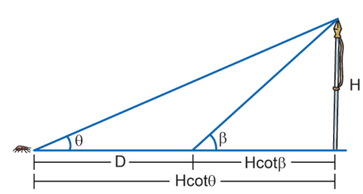
$$\frac{1}{4} = \frac{H}{36 + x} \Rightarrow 36 + x = 4H$$

$$36 + x = 4(21)$$

$\therefore x = 48$ m

Clave E

20.



Del gráfico: $H\cot\theta = H\cot\beta + D$

$H(\cot\theta - \cot\beta) = D$

$\therefore H = \dfrac{D}{\cot\theta - \cot\beta}$

Clave B

Nivel 3 (página 56) Unidad 3

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Del gráfico:

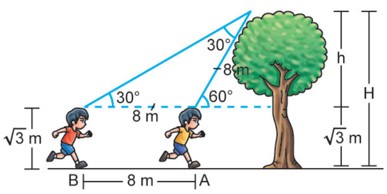
$H = 24\tan37° + 24\tan45°$

$H = 24(\frac{3}{4}) + 24\ (1)$

$\therefore\ H = 42$ m

Clave C

24.



Del gráfico:

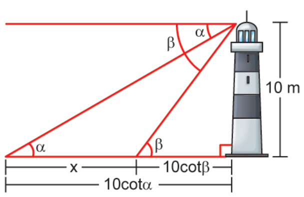
$h = 8\text{sen}60°$

$h = 4\sqrt{3}$ m $\qquad \therefore H = 4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$ m

Clave E

Resolución de problemas

25.



Del gráfico:

$x + 10\cot\beta = 10\cot\alpha$

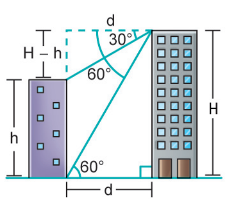
$x = 10\cot\alpha - 10\cot\beta$

$x = 10\underbrace{(\cot\alpha - \cot\beta)}_{4\ (\text{dato})}$

$\therefore x = 40$ m

Clave C

26.



Del gráfico: $H = d\tan60°$

$\underbrace{H}_{d\tan60°} - h = d\tan30°$

$\Rightarrow h = d(\tan60° - \tan30°)$

Piden: $\dfrac{H}{h} = \dfrac{d\tan60°}{d(\tan60° - \tan30°)}$

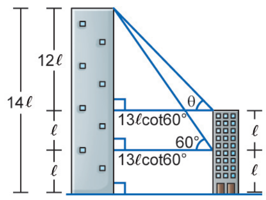
$\dfrac{H}{h} = \dfrac{\tan60°}{\tan60° - \tan30°}$

$\dfrac{H}{h} = \dfrac{\sqrt{3}}{\sqrt{3} - \dfrac{\sqrt{3}}{3}} = \dfrac{3\sqrt{3}}{2\sqrt{3}} = \dfrac{3}{2} = 1{,}5$

$\therefore \dfrac{H}{h} = 1{,}5$

Clave B

27.



Sea H la altura del edificio, entonces, del dato:

$\dfrac{2\ell}{H} = \dfrac{1}{7} \Rightarrow H = 14\ell$

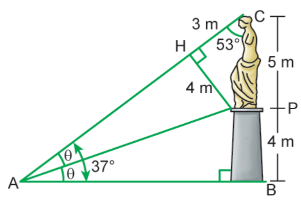
Del gráfico:

$\tan\theta = \dfrac{12\ell}{13\ell\cot60°} = \dfrac{12}{13\underbrace{\cot60°}_{\frac{\sqrt{3}}{3}}} = \dfrac{12.3}{13.\sqrt{3}}$

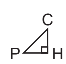
$\therefore \tan\theta = \dfrac{12\sqrt{3}}{13}$

Clave B

28.



PH = 4, por el teorema de la bisectriz interior.



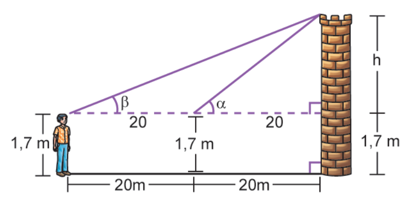
notable de 37° y 53°:

$AB = 9\cot37° = 9\left(\dfrac{4}{3}\right) = 12$

$\therefore AB = 12$ m

Clave E

29.



Del gráfico: $h = 20\tan\alpha = 40\tan\beta \qquad$ ...(I)

$\Rightarrow \tan\alpha = 2\tan\beta$

Del dato: $\tan\alpha + \tan\beta = 0{,}75$

$2\tan\beta + \tan\beta = 0{,}75$

$\Rightarrow 3\tan\beta = 0{,}75$

$\tan\beta = 0{,}25$

En (I): $h = 40\tan\beta$

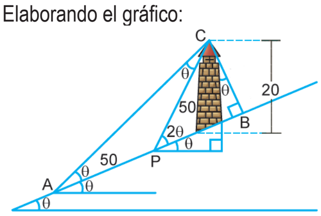
$= 40(0{,}25) = 10$

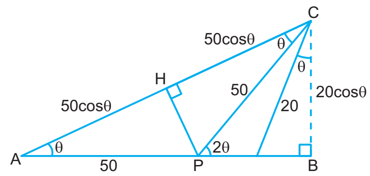
Entonces, la altura de la torre será:

$h + 1{,}7 = 10 + 1{,}7 = 11{,}7$

Clave B

30. Elaborando el gráfico:





En el triángulo rectángulo ABC:

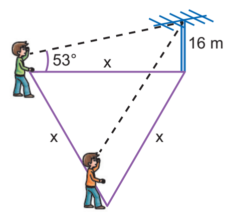
$\text{sen}\theta = \dfrac{20\cos\theta}{100\cos\theta} = \dfrac{20}{100} = \dfrac{1}{5}$

$\therefore \text{sen}\theta = \dfrac{1}{5}$

Clave D

MARATÓN MATEMÁTICA (página 58)

1. Del gráfico tenemos:

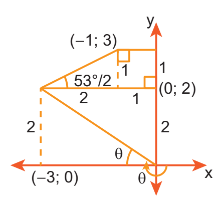


$x = (16\text{ m})(\cot53°) \Rightarrow x = (16\text{m})\left(\dfrac{3}{4}\right) = 12$ m

$\therefore\ x = 12$ m

Clave A

2. Del gráfico tenemos:

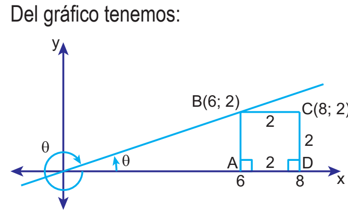


$\tan\theta = \dfrac{2}{-3} = \dfrac{-2}{3}$

$\therefore\ 3\tan\theta = -2$

Clave B

3. Del gráfico tenemos:



$\tan\theta = \dfrac{6}{2} = \dfrac{1}{3}$

Clave D

4. Tenemos:

L_1: $(a-2)x - 3y + 8 = 0$ y L_2: $(2-2a)x + (2a+1)y + 6 = 0$

$\Rightarrow M_1 = \dfrac{-(a-2)}{(-3)} \wedge M_2 = \dfrac{-(2)-2a}{2a+1}$

Sabemos por dato:

$m_1 = m_2 \Rightarrow \dfrac{a-2}{3} = \dfrac{2a-2}{2a+1}$

$2a^2 - 3a - 2 = 6a - 6$

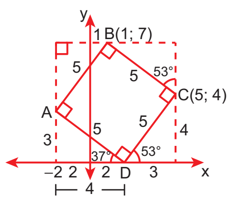
$2a^2 - 9a + 4 = 0$

$a = \dfrac{1}{2} \wedge a = 4$

$(a > 1) \therefore a = 4$

Clave C

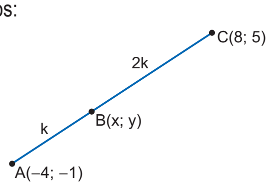
5.



$A = (-2; 3) \Rightarrow$ ordenada $A = 3$

Clave B

6. Tenemos:



Sabemos:

$x = \dfrac{k(8) + 2k(-4)}{k+2k} = \dfrac{8k-8k}{3k} = 0$

$y = \dfrac{k(5) + 2k(-1)}{k+2k} = \dfrac{5k-2k}{3k} = 1$

$\therefore B = (0; 1)$

Clave E

7. Notamos que B es punto medio de $\overline{AC}$.

$\Rightarrow B(-1; 2) = \dfrac{A(x, y) + C(2, 5)}{2}$

$-1 = \dfrac{x+2}{2} \Rightarrow -2 = x + 2$

$x = -4$

$2 = \dfrac{y+5}{2} \Rightarrow 4 = y + 5$

$y = -1$

$\Rightarrow A(x; y) = (-4; -1)$

Nos piden:

$\tan\theta = \dfrac{y}{x} = \dfrac{-1}{-4}$

$\therefore \tan\theta = \dfrac{1}{4}$

Clave C

8. De la ecuación tenemos:

$25\text{sen}^2\theta + 10\text{sen}\theta - 8 = 0$

$5\text{sen}\theta \qquad 4 \Rightarrow (5\text{sen}\theta + 4) = 0$

$5\text{sen}\theta \qquad -2 \quad (5\text{sen}\theta - 2) = 0$

Como $\theta \in$ IIIC:

$\Rightarrow \text{sen}\theta = \dfrac{-4}{5}$

Luego:

$\text{sen}^2\theta + \cos^2\theta = 1$

$\left(\dfrac{-4}{5}\right)^2 + \cos^2\theta = 1 \Rightarrow \cos^2\theta = \dfrac{9}{25}$

$(\theta \in \text{IIIC}) \cos\theta = \dfrac{-3}{5}$

$\therefore \cos\theta + 1 = \dfrac{2}{5}$

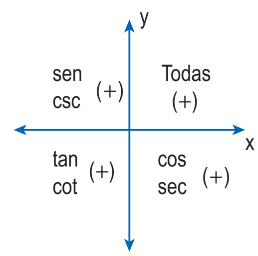
Clave C

9. De los datos:

$\cos\theta > 0 \quad (+)$

$\tan\theta < 0 \quad (-)$

$\Rightarrow \text{sen}\theta < 0 \ (-)$



$\therefore \theta \in$ IVC

Clave A

REDUCCIÓN AL PRIMER CUADRANTE

APLICAMOS LO APRENDIDO (página 61) Unidad 4

1. $M = \dfrac{(\sec 60°) - (-\cos 45°)}{(\text{sen}45°) - (-\csc 30°)}$

$M = \dfrac{(2) + \left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right) + (2)} = \dfrac{4+\sqrt{2}}{4+\sqrt{2}} = 1$

$\therefore M = 1$

Clave E

2. $E = \dfrac{(\text{sen}60°)(-\tan 45°)}{(-\text{sen}60°)(\cos 60°)}$

$E = \dfrac{\tan 45°}{\cos 60°} = \dfrac{(1)}{\left(\frac{1}{2}\right)} = 2$

$\therefore E = 2$

Clave A

3. $\text{sen}(180° - 30°) + 2\cos(180° + 30°) + \tan(360° - 60°) + \text{sen}(360° - 30°)$

$= \text{sen}30° + (-2\cos30°) + (-\tan60°) + (-\text{sen}30°)$

$= \text{sen}30° - 2\cos30° - \tan60° - \text{sen}30°$

$= -2\left(\frac{\sqrt{3}}{2}\right) - (\sqrt{3})$

$= -\sqrt{3} - \sqrt{3} = -2\sqrt{3}$

Clave C

4. $\tan(180° - 30°) + \tan(180° - 45°) + \tan(180° - 60°)$

$= (-\tan30°) + (-\tan45°) + (-\tan60°)$

$= -\frac{\sqrt{3}}{3} - 1 - \sqrt{3}$

$= \dfrac{-3\sqrt{3} - \sqrt{3}}{3} - 1 = \dfrac{-4\sqrt{3}}{3} - 1$

$= \dfrac{-4\sqrt{3} - 3}{3}$

Clave C

5. $[\text{sen}(360° + 45°)]^2 + [\cos(360° + 120°)]^2$

$= [\text{sen}45°]^2 + [\cos120°]^2$

$= \text{sen}^2 45° + (-\cos60°)^2$

$= \text{sen}^2 45° + \cos^2 60° = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\therefore \text{sen}^2(405°) + \cos^2(480°) = \frac{3}{4}$

Clave C

6. $M = [\text{sen}(360° \times 9 + 120°)]^2 . [\cos(360° \times 5 + 150°)]^3$

$M = (\text{sen}120°)^2(\cos150°)^3$

$M = (\text{sen}60°)^2\,(-\cos30°)^3$

$M = \left(\frac{\sqrt{3}}{2}\right)^2\left(-\frac{\sqrt{3}}{2}\right)^3 = \left(\frac{3}{4}\right)\cdot\left(-\frac{3\sqrt{3}}{8}\right)$

$\therefore M = -\frac{9\sqrt{3}}{32}$

Clave B

7. $\underbrace{\tan(180° + x)}_{+\,\tan x} = -\tan x$ ...(F)

$\underbrace{\cos(360° - x)}_{+\,\cos x} = -\cos x$ ...(F)

$\underbrace{\text{sen}(360° - x)}_{-\,\text{sen}x} = -\text{sen}x$ ...(V)

$\therefore$ FFV

Clave C

8. $Q = \dfrac{-\cot x - (+\tan x)}{+\csc y + (+\text{sen}y)} = \dfrac{-\cot x - \tan x}{\csc y + \text{sen}y}$

$\therefore Q = \dfrac{-\tan x - \cot x}{\csc y + \text{sen}y}$

Clave B

9. $Q = -\text{sen}45°\,[\cos30° - (-\tan60°)]$

$Q = -\frac{\sqrt{2}}{2}\left[\frac{\sqrt{3}}{2} + \sqrt{3}\right]$

$Q = -\frac{\sqrt{2}}{2}\left(\frac{3\sqrt{3}}{2}\right) = -\frac{3\sqrt{6}}{4}$

$\therefore Q = -\frac{3\sqrt{6}}{4}$

Clave E

10. $E = 3\tan(180° + 45°) - 4\cos(180° - 60°) + 3\cot(180° - 45°)$

$E = 3(\tan45°) - 4(-\cos60°) + 3(-\cot45°)$

$E = 3\tan45° + 4\cos60° - 3\cot45°$

$E = 3(1) + 4\left(\frac{1}{2}\right) - 3(1) = 3 + 2 - 3 = 2$

$\therefore E = 2$

Clave E

11. $M = 6\sqrt{3}\,\cot(270° - 30°)$

$M = 6\sqrt{3}\,\tan30°$

$M = 6\sqrt{3}\,\frac{\sqrt{3}}{3}$

$M = 6$

Clave B

12. $N = 3\sqrt{3} - 2\cos(90° + 60°)$

$N = 3\sqrt{3} - 2(-\text{sen}60°)$

$N = 3\sqrt{3} + 2\left(\frac{\sqrt{3}}{2}\right)$

$N = 4\sqrt{3}$

Clave A

13. $E = \text{sen}(90° + 10°) . \cos(180° + 10°)$

$E = (\cos10°) . (-\cos10°)$

$E = -\cos^2 10°$

$E = -a^2$

$\therefore E = -a^2$

Clave E

14. $\csc(90 - A) - x\cos A . \cot(90 - A) = \text{sen}(90 - A)$

$\sec A - x\cos A . \tan A = \cos A$

$\sec A - x\cos A . \frac{\text{sen}A}{\cos A} = \cos A$

$\sec A - x\text{sen}A = \cos A$

$\frac{1}{\cos A} - \cos A = x\text{sen}A$

$\frac{1 - \cos^2 A}{\cos A} = x\text{sen}A \quad \Rightarrow \quad x = \tan A$

Clave C

PRACTIQUEMOS

Nivel 1 (página 63) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $\text{sen}1560° = \text{sen}(360° \times 4 + 120°) = \text{sen}120°$

$\text{sen}120° = \text{sen}60° = \frac{\sqrt{3}}{2}$

$\therefore \text{sen}1560° = \frac{\sqrt{3}}{2}$

Clave C

4. $\tan1230° = \tan(360° \times 3 + 150°)$

$\tan150° = \tan(180° - 30°)$

$\tan1230° = -\tan30° = -\frac{\sqrt{3}}{3}$

$\therefore \tan1230° = -\frac{\sqrt{3}}{3}$

Clave D

5. $\tan855° = \tan(360° \times 2 + 135°) = \tan135°$

$\tan135° = -\tan45° = -(1)$

$\therefore \tan855° = -1$

Clave D

6. $\sec2360° = \sec(360° \times 6 + 200°) = \sec200°$

$\sec200° = \sec(180° + 20°) = -\sec20°$

$\therefore \sec2360° = -\sec20°$

Clave E

7. $\tan2870° = \tan(360° \times 7 + 350°) = \tan350°$

$\tan350° = \tan(360° - 10°) = -\tan10°$

$\therefore \tan2870° = -\tan10°$

Clave D

8. $\tan3540° = \tan(360° \times 9 + 300°) = \tan300°$

$\tan300° = \tan(360° - 60°)$

$= -\tan60° = -\sqrt{3}$

$\therefore \tan3540° = -\sqrt{3}$

Clave E

9. $\sec 4650° = \sec(360° \times 12 + 330°)$
$= \sec 330°$
$\sec 330° = \sec(360° - 30°)$
$= \sec 30° = \frac{2\sqrt{3}}{3}$
$\therefore \sec 4650° = \frac{2\sqrt{3}}{3}$

Clave D

10. $\tan(-300°) = -\tan(300°)$
$-\tan(300°) = -\tan(360° - 60°)$
$= -(-\tan 60°)$
$= \tan 60° = \sqrt{3}$
$\therefore \tan(-300°) = \sqrt{3}$

Clave B

Nivel 2 (página 63) Unidad 4

Comunicación matemática

11.

12.

Razonamiento y demostración

13. $A = 2 - 10\text{sen}330°$
$A = 2 - 10\text{sen}(360° - 30°)$
$A = 2 - 10(-\text{sen}30°) = 2 + 10\text{sen}30°$
$A = 2 + 10\left(\frac{1}{2}\right) = 2 + 5 = 7$
$\therefore A = 7$

Clave D

14. $E = 4\sqrt{3}\,(\cos 210°)$
$E = 4\sqrt{3}\,[\cos(180° + 30°)]$
$E = 4\sqrt{3}\,(-\cos 30°)$
$E = -4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = -2 \,.\, 3 = -6$
$\therefore E = -6$

Clave E

15. $S = \sqrt{2}\cos 315°$
$S = \sqrt{2}\cos(360° - 45°)$
$S = \sqrt{2}\,(\cos 45°)$
$S = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{2}{2} = 1$
$\therefore S = 1$

Clave A

16. $E = 2 - \tan 135°$
$E = 2 - \tan(180° - 45°)$
$E = 2 - (-\tan 45°)$
$E = 2 + \underbrace{\tan 45°}_{1} = 2 + 1 = 3$
$\therefore E = 3$

Clave C

17. $E = \cot 135° + \tan 135°$
$E = \cot(180° - 45°) + \tan(180° - 45°)$
$E = (-\cot 45°) + (-\tan 45°)$
$E = (-1) + (-1) = -2$
$\therefore E = -2$

Clave C

18. $R = \text{sen}120° + \cos 210°$
$R = \text{sen}(180° - 60°) + \cos(180° + 30°)$
$R = (+\text{sen}60°) + (-\cos 30°)$
$R = \text{sen}60° - \cos 30°$
$R = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$
$\therefore R = 0$

Clave C

19. $E = \sqrt{2} + \sec 225°$
$E = \sqrt{2} + \sec(180° + 45°)$
$E = \sqrt{2} + (-\sec 45°)$
$E = \sqrt{2} - \underbrace{\sec 45°}_{\sqrt{2}} = \sqrt{2} - \sqrt{2} = 0$
$\therefore E = 0$

Clave C

20. $M = 3 - \sec 3000°$
$M = 3 - \sec(360° \times 8 + 120°)$
$M = 3 - \sec 120°$
$M = 3 - (-\sec 60°) = 3 + \underbrace{\sec 60°}_{2}$
$M = 3 + 2 = 5$
$\therefore M = 5$

Clave D

Nivel 3 (página 64) Unidad 4

Comunicación matemática

21.

22.

Razonamiento y demostración

23. $R = 1 + 8(\cos 405°)^2$
$R = 1 + 8(\cos(360° + 45°))^2$
$R = 1 + 8(\cos 45°)^2$
$R = 1 + 8\left(\frac{\sqrt{2}}{2}\right)^2 = 1 + 8\left(\frac{2}{4}\right)$
$R = 1 + 2 \times 2 = 5$
$\therefore R = 5$

Clave A

24. $S = 4 - 6\sqrt{3}\,(\text{sen}300°)$
$S = 4 - 6\sqrt{3}\,(\text{sen}(360° - 60°))$
$S = 4 - 6\sqrt{3}\,(-\text{sen}60°) = 4 + 6\sqrt{3}\,\text{sen}60°$
$S = 4 + 6\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 4 + \frac{6 \times 3}{2}$
$\therefore S = 13$

Clave E

25. $M = -\sqrt{3}\,[\csc(-120°)]$
$M = -\sqrt{3}\,(-\csc 120°) = \sqrt{3}\csc 120°$
$M = \sqrt{3}\,.\,\csc 60° = \sqrt{3}\left(\frac{2\sqrt{3}}{3}\right) = \frac{3.2}{3} = 2$
$\therefore M = 2$

Clave B

26. $M = 8(\text{sen}120°)^2 - 1$
$M = 8(\text{sen}(180° - 60°))^2 - 1$
$M = 8(\text{sen}60°)^2 - 1$
$M = 8\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 8\left(\frac{3}{4}\right) - 1 = 6 - 1 = 5$
$\therefore M = 5$

Clave A

27. $T = \sqrt{3}\,(4\sqrt{3} - \tan 300°)$
$T = \sqrt{3}\,(4\sqrt{3} - \tan(360° - 60°))$
$T = \sqrt{3}\,(4\sqrt{3} - (-\tan 60°))$
$T = \sqrt{3}\,(4\sqrt{3} + \underbrace{\tan 60°}_{\sqrt{3}}) = \sqrt{3}\,(4\sqrt{3} + \sqrt{3})$
$T = \sqrt{3}\,(5\sqrt{3}) = 5 \times 3 = 15$
$\therefore T = 15$

Clave D

28. $A = \sqrt{8\sqrt{3}\,(\sec 330°)}$
$\sec 330° = \sec(360° - 30°)$
$\sec 330° = \sec 30° = \frac{2\sqrt{3}}{3}$
$A = \sqrt{8\sqrt{3}\left(\frac{2\sqrt{3}}{3}\right)} = \sqrt{\frac{16.3}{3}} = \sqrt{16} = 4$
$\therefore A = 4$

Clave D

29. $R = 6\sqrt{3}\,[\sec(-210°)]$
$R = 6\sqrt{3}\,(\sec 210°) = 6\sqrt{3}\sec 210°$
$R = 6\sqrt{3}\sec(180° + 30°) = 6\sqrt{3}\,(-\sec 30°)$
$R = -6\sqrt{3}\sec 30° = -6\sqrt{3} \times \frac{2\sqrt{3}}{3}$
$R = -\frac{6 \times 2 \times 3}{3} = -12$

Clave D

30. $(\csc 150°)^{2x-6} = 256$
$(\csc 30°)^{2x-6} = 256$
$(2)^{2x-6} = (2)^8$
$\Rightarrow 2x - 6 = 8$
$2x = 14$
$\therefore x = 7$

Clave E

APLICAMOS LO APRENDIDO (página 65) Unidad 4

1. $L = \cos^2\alpha\underbrace{(1+\tan^2\alpha)}_{\text{identidad}}$

$L = \underbrace{\cos^2\alpha \,.\, \sec^2\alpha}_{\text{recíprocas}} = 1$

$\therefore L = 1$

Clave C

2. Por factor común tenemos:

$C = \sqrt{\dfrac{\csc^2x(\sec^2x-1)}{\sec^2x(\csc^2x-1)}} \Rightarrow \begin{array}{l}\tan^2x = \sec^2x - 1\\ \cot^2x = \csc^2x - 1\end{array}$

Reemplazamos en C:

$C = \sqrt{\dfrac{\csc^2x(\tan^2x)}{\sec^2x(\cot^2x)}} = \sqrt{\dfrac{\csc^2x \,.\, \text{sen}^2x \,.\, \sec^2x}{\sec^2x \,.\, \cos^2x \,.\, \csc^2x}}$

$C = \sqrt{\tan^2x}\,;\, x \in \text{IC}$

$C = \tan x$

Clave B

3. Sabemos: $\sec^2x = \tan^2x + 1$; reemplazamos

$\cos^2x + \tan^2x = 1$

$\cos^2x + \tan^2x + 1 = 1 + 1$

$\cos^2x + \sec^2x = 2$

$\cos^2x + \sec^2x + \underbrace{2\sec x \,.\, \cos x}_{1} = 2 + 2$

$(\cos x + \sec x)^2 = 4$

$\cos x + \sec x = \sqrt{4}$

$\cos x + \sec x = 2$

Clave A

4. Simplificamos el dato:

$2\,\text{sen}\alpha - \cos\alpha = 0$

$2\,\text{sen}\alpha = \cos\alpha$

$2 = \dfrac{\cos\alpha}{\text{sen}\alpha} = \cot\alpha = 2$

Nos piden:

$\csc^2\alpha = 1 + \cot^2\alpha$

$\csc^2\alpha = 1 + (2)^2$

$\csc^2\alpha = 5$

Clave B

5. $A = \dfrac{\text{sen}x}{\left(\dfrac{1}{\text{sen}x}\right)} + \dfrac{\cos x}{\left(\dfrac{1}{\cos x}\right)}$

$A = \text{sen}x(\text{sen}x) + \cos x(\cos x)$

$A = \text{sen}^2x + \cos^2x$

$\therefore A = 1$

Clave E

6. $A = \dfrac{(1-\text{sen}^2x)}{1+\text{sen}x} - \dfrac{(1-\cos^2x)}{1+\cos x}$

$A = \dfrac{(1+\text{sen}x)(1-\text{sen}x)}{(1+\text{sen}x)} - \dfrac{(1+\cos x)(1-\cos x)}{(1+\cos x)}$

$A = (1-\text{sen}x) - (1-\cos x)$

$A = 1 - \text{sen}x - 1 + \cos x$

$\therefore A = \cos x - \text{sen}x$

Clave B

7. $\underbrace{\tan x} + \underbrace{\cot x} = 3$ (dato)

$\dfrac{\text{sen}x}{\cos x} + \dfrac{\cos x}{\text{sen}x} = 3$

$\dfrac{\text{sen}^2x + \cos^2x}{\text{sen}x \,.\, \cos x} = 3$

$\Rightarrow \underbrace{\text{sen}^2x + \cos^2x}_{1} = 3(\text{sen}x \,.\, \cos x)$

$\Rightarrow \text{sen}x \,.\, \cos x = \dfrac{1}{3}$

Piden:

$Q = \text{sen}x(\csc x + \cos x) + \csc x(\text{sen}x + \sec x)$

$Q = \underbrace{\text{sen}x\csc x}_{1} + \text{sen}x\cos x + \underbrace{\csc x\,\text{sen}x}_{1} + \csc x\sec x$

$Q = 1 + \text{sen}x\cos x + 1 + \dfrac{1}{\text{sen}x\cos x}$

$Q = 2 + \text{sen}x\cos x + \dfrac{1}{\text{sen}x\cos x}$

$Q = 2 + \left(\dfrac{1}{3}\right) + \dfrac{1}{\left(\dfrac{1}{3}\right)} = 5 + \dfrac{1}{3}$

$Q = \dfrac{16}{3}$

Clave E

8. Desarrollamos la expresión:

$(4\text{sen}x + \cos x)^2 + (\text{sen}x + 4\cos x)^2 = m + n\text{sen}x \,.\, \cos x$

$16\text{sen}^2x + \cos^2x + 8\text{sen}x\cos x + \text{sen}^2x + 16\cos^2x + 8\text{sen}x\cos x = m + n\text{sen}x\cos x$

$17\text{sen}^2x + 17\cos^2x + 16\text{sen}x \,.\, \cos x = m + n\text{sen}x \,.\, \cos x$

$17(\text{sen}^2x + \cos^2x) + 16\text{sen}x \,.\, \cos x = m + n\text{sen}x \,.\, \cos x$

$17(1) + 16\text{sen}x \,.\, \cos x = m + n\,\text{sen}x \,.\, \cos x$

$\Rightarrow m = 17 \;\wedge\; n = 16$

$\therefore m - n = 1$

Clave E

9. Desarrollamos la expresión:

$L = \text{sen}\theta(\csc\theta + \text{sen}\theta) + \cos\theta(\sec\theta + \cos\theta) - 2$

$L = \text{sen}\theta \,.\, \csc\theta + \text{sen}^2\theta + \cos\theta \,.\, \sec\theta + \cos^2\theta - 2$

$L = 1 + \underbrace{\text{sen}^2\theta + 1 + \cos^2\theta}_{1} - 2$

$L = 1 + 1 + 1 - 2 \Rightarrow L = 1$

Clave C

10. Desarrollamos la expresión:

$D = \dfrac{(\text{sen}^2x + \cos^2x + 2\text{sen}x \,.\, \cos x - 1)\csc x}{2\cos x}$

$D = \dfrac{(1 + 2\text{sen}x \,.\, \cos x - 1)\csc x}{2\cos x}$

$D = \dfrac{2\text{sen}x \,.\, \cos x \,.\, \csc x}{2\cos x}$

$D = \text{sen}x \,.\, \csc x \Rightarrow D = 1$

Clave A

11. $\dfrac{\tan^2x - \text{sen}^2x}{\tan^2x} = M \Rightarrow \underbrace{\dfrac{\tan^2x}{\tan^2x}} - \dfrac{\text{sen}^2x}{\tan^2x} = M$

$1 - \dfrac{\text{sen}^2x}{\left(\dfrac{\text{sen}^2x}{\cos^2x}\right)} = M$

$\Rightarrow M = \underbrace{1 - \cos^2x}$

$\therefore M = \text{sen}^2x$

Clave D

12. $\dfrac{\tan^2x - 1}{\dfrac{1}{\tan^2x} - 1} + \sec^2x \Rightarrow \dfrac{\tan^2x - 1}{\dfrac{1-\tan^2x}{\tan^2x}} + \sec^2x$

$\dfrac{\tan^2x - 1}{\dfrac{1}{\dfrac{\tan^2x - 1}{-\tan^2x}}} + \sec^2x \Rightarrow \underbrace{-\tan^2x + \sec^2x}_{1}$

Clave C

13. $E = \csc x + \csc x \,.\, \cos x - \cot x - \cot x \,.\, \cos x$

$E = \csc x + \dfrac{\cos x}{\text{sen}x} - \cot x - \left(\dfrac{\cos x}{\text{sen}x}\right).\cos x$

$E = \dfrac{1}{\text{sen}x} + \cot x - \cot x - \dfrac{\cos^2x}{\text{sen}x}$

$E = \dfrac{1}{\text{sen}x} - \dfrac{\cos^2x}{\text{sen}x} = \dfrac{1-\cos^2x}{\text{sen}x} = \dfrac{\text{sen}^2x}{\text{sen}x} = \text{sen}x$

$\therefore E = \text{sen}x$

Clave B

14. Elevamos al cuadrado la expresión:

$(\text{sen}x + \cos x)^2 = \left(\dfrac{\sqrt{5}}{2}\right)^2$

$\text{sen}^2x + \cos^2x + 2\text{sen}x \,.\, \cos x = \dfrac{5}{4}$

$1 + 2\text{sen}x \,.\, \cos x = \dfrac{5}{4}$

$\Rightarrow 2\text{sen}x \,.\, \cos x = \dfrac{1}{4}$

$\text{sen}x \,.\, \cos x = \dfrac{1}{8}$

Clave B

PRACTIQUEMOS

Nivel 1 (página 67) Unidad 4

Comunicación matemática

1.

2.

A) $2\text{sen}^2\alpha + 2\cos^2\alpha = 1$
$2(\text{sen}^2\alpha + \cos^2\alpha) = 1$ (F)

B) $\tan\alpha \,.\, \cot\alpha = \dfrac{\text{sen}\alpha \,.\, \csc\alpha}{\cos\alpha \,.\, \sec\alpha}$ (V)

C) $\cot\alpha\ .\ \text{sen}\alpha = \cos\alpha$

$\cot\alpha = \dfrac{\cos\alpha}{\text{sen}\alpha}$ (V)

D) $\sec^2\alpha - 2\tan^2\alpha = 1$

$\sec^2\alpha - \tan^2\alpha = 1$ (F)

E) $\tan\alpha + \cot\alpha = \sec\alpha\ .\ \csc\alpha$

$\dfrac{\text{sen}\alpha}{\cos\alpha} + \dfrac{\cos\alpha}{\text{sen}\alpha} = \sec\alpha\ .\ \csc\alpha$

$\dfrac{\text{sen}^2\alpha + \cos^2\alpha}{\cos\alpha\ .\ \text{sen}\alpha} = \sec\alpha\ .\ \csc\alpha$

$\dfrac{1}{\cos\alpha} \cdot \dfrac{1}{\text{sen}\alpha} = \sec\alpha\ .\ \csc\alpha$ (V)

Razonamiento y demostración

3. $A = (3\text{sen}x + \cos x)^2 + (\text{sen}x - 3\cos x)^2$

$A = 9\text{sen}^2x + \cos^2x + 6\text{sen}x\cos x + \text{sen}^2x + 9\cos^2x - 6\text{sen}x\cos x$

$A = 10\text{sen}^2x + 10\cos^2x$

$A = 10\underbrace{(\text{sen}^2x + \cos^2x)}_{1} = 10$

$\therefore A = 10$

Clave B

4. $L = \sec^2x\ .\ \cot x\ .\ \text{sen}x$

$L = \sec^2x\ .\ \dfrac{\cos x}{\text{sen}x}\ \text{sen}x = \sec^2x\ .\ \cos x$

$L = \sec x\ .\ \underbrace{\sec x\ .\ \cos x}_{1} = \sec x$

$\therefore L = \sec x$

Clave C

5. $C = \dfrac{(\text{sen}x + \cos x + 1)(\text{sen}x + \cos x - 1)}{\text{sen}x\cos x}$

$C = \dfrac{(\text{sen}x + \cos x)^2 - 1^2}{\text{sen}x\cos x}$

$C = \dfrac{\text{sen}^2x + \cos^2x + 2\text{sen}x\cos x - 1}{\text{sen}x\cos x}$

$C = \dfrac{1 - 1 + 2\text{sen}x\cos x}{\text{sen}x\cos x} = \dfrac{2\text{sen}x\cos x}{\text{sen}x\cos x} = 2$

$\therefore C = 2$

Clave B

6. $D = \sec^2x\ .\ \csc^2x - (\tan x - \cot x)^2$

$D = \sec^2x\ .\ \csc^2x - (\tan^2x - \underbrace{2\tan x\cot x}_{1} + \cot^2x)$

$D = \sec^2x + \csc^2x - \tan^2x - \cot^2x + 2$

$D = 1 + 1 + 2 \quad \therefore D = 4$

Clave D

7. $L = \dfrac{\sec x.\csc x + \tan x}{\tan x}$

$L = \dfrac{\sec x.\csc x}{\tan x} + \dfrac{\tan x}{\tan x}$

$L = \dfrac{\sec x.\csc x}{\dfrac{(\text{sen}x)}{(\cos x)}} + 1 = \dfrac{\overbrace{\sec x.\cos x}^{1}.\csc x}{\text{sen}x} + 1$

$\Rightarrow L = \dfrac{\csc x}{\text{sen}x} + 1 = \csc^2x + 1 \quad \therefore L = \csc^2x + 1$

Clave B

8. $U = \dfrac{(\text{sen}x - 1)^2 + (\cos x - 1)^2 - 1}{1 - \text{sen}x - \cos x}$

$U = \dfrac{(\text{sen}^2x - 2\text{sen}x + 1) + (\cos^2x - 2\cos x + 1) - 1}{1 - \text{sen}x - \cos x}$

$U = \dfrac{(\text{sen}^2x + \cos^2x - 1) + (2 - 2\text{sen}x - 2\cos x)}{1 - \text{sen}x - \cos x}$

$U = \dfrac{2(1 - \text{sen}x - \cos x)}{(1 - \text{sen}x - \cos x)} = 2 \qquad \therefore U = 2$

Clave B

9. $L = (\text{sen}^2x - \cos^2x)^2 + 4\text{sen}^2x\cos^2x$

$L = \text{sen}^4x - 2\text{sen}^2x\cos^2x + \cos^4x + 4\text{sen}^2x\cos^2x$

$L = \text{sen}^4x + 2\text{sen}^2x\cos^2x + \cos^4x$

$L = \underbrace{(\text{sen}^2x + \cos^2x)^2}_{(1)^2}$

$\therefore L = 1$

Clave A

10. $A = \dfrac{\sec\theta - \cos\theta}{\csc\theta - \text{sen}\theta} = \dfrac{\dfrac{1}{\cos\theta} - \cos\theta}{\dfrac{1}{\text{sen}\theta} - \text{sen}\theta}$

$A = \dfrac{\dfrac{(1 - \cos^2\theta)}{\cos\theta}}{\dfrac{(1 - \text{sen}^2\theta)}{\text{sen}\theta}} = \dfrac{(\text{sen}^2\theta)\text{sen}\theta}{(\cos^2\theta)\cos\theta} = \dfrac{\text{sen}^3\theta}{\cos^3\theta}$

$A = \left(\dfrac{\text{sen}\theta}{\cos\theta}\right)^3 = \tan^3\theta$

$\therefore A = \tan^3\theta$

Clave E

Resolución de problemas

11. Operamos la expresión:

$\csc x\ .\ \tan x\ .\ \cos^2x - \dfrac{\cot x}{\csc x} = a\text{sen}x$

$\dfrac{1}{\text{sen}x} \cdot \dfrac{\text{sen}x}{\cos x} \cdot \cos^2x - \dfrac{\dfrac{\cos x}{\text{sen}x}}{\dfrac{1}{\text{sen}x}} = a\text{sen}x$

$\cos x - \cos x = a\text{sen}x$

$0 = a\text{sen}x$

$a = 0$

$\therefore\ a^2 + 1 = 1$

Clave D

12. De la expresión, tenemos:

$(M\sec x - \cos x)(\csc x - \text{sen}x) = \text{sen}x\ .\ \cos x$

$M\sec x\ .\ \csc x - M\sec x\ .\ \text{sen}x - \cos x\ .\ \csc x + \cos x\ .\ \text{sen}x = \text{sen}x\ .\ \cos x$

$\dfrac{M}{\text{sen}x\ .\ \cos x} = M\dfrac{\text{sen}x}{\cos x} + \dfrac{\cos x}{\text{sen}x}$

$\dfrac{M}{\text{sen}x\ .\ \cos x} = \dfrac{M\text{sen}^2x + \cos^2x}{\text{sen}x\ .\ \cos x}$

$M = M\text{sen}^2x + \cos^2x$

$M(1 - \text{sen}^2x) = \cos^2x$

$\therefore\ M = 1$

Clave C

Nivel 2 (página 67) Unidad 4

Comunicación matemática

13. • $\tan x\ .\ \cos x = \text{sen}x$

$\dfrac{\text{sen}x}{\cos x} . \cos x = \text{sen}x$ (V)

• $\tan x + \cot x = \csc x$

$\dfrac{\text{sen}x}{\cos x} + \dfrac{\cos x}{\text{sen}x} = \csc x$

$\dfrac{\text{sen}^2x + \cos^2x}{\cos x + \text{sen}x} \neq \csc x$ (F)

• $\text{sen}^3x\ .\ \csc x + \cos^3x\ .\ \sec x = 1$

$\text{sen}^3x\ .\ \dfrac{1}{\text{sen}x} + \cos^3x . \dfrac{1}{\cos x} = 1$

$\text{sen}^2x + \cos^2x = 1$ (V)

• $\text{sen}^4\theta + \cos^4\theta = 1 - 3\text{sen}^2\theta\ .\ \cos^2\theta$

$1 - 2\text{sen}^2\theta\ .\ \cos^2\theta \neq 1 - 3\text{sen}^2\theta\ .\ \cos^2\theta$ (F)

$\therefore$ II y IV son falsas

Clave D

14.

15. $C = \tan^2x\ .\ \cos x\ .\ \csc x$

$C = \tan^2x\ .\ \dfrac{\cos x}{\text{sen}x} = \tan^2x\ .\ \cot x$

$C = \tan x\ .\ \underbrace{\tan x\ .\ \cot x}_{1} = \tan x$

$\therefore C = \tan x$

Clave B

16. $I = \dfrac{\text{sen}x.\sec x.\tan x + \text{sen}^2x.\sec^2x}{\sec^2x - 1}$

$I = \dfrac{\text{sen}x\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\text{sen}x}{\cos x}\right) + \text{sen}^2x\left(\dfrac{1}{\cos^2x}\right)}{\tan^2x}$

$I = \dfrac{\dfrac{\text{sen}^2x}{\cos^2x} + \dfrac{\text{sen}^2x}{\cos^2x}}{\tan^2x} = \dfrac{2\left(\dfrac{\text{sen}x}{\cos x}\right)^2}{\tan^2x} = \dfrac{2\tan^2x}{\tan^2x}$

$\therefore I = 2$

Clave B

17. $U = \dfrac{\tan x.\cos x + \text{sen}^2x.\csc x}{1 - \cos^2x}$

$U = \dfrac{\left(\dfrac{\text{sen}x}{\cos x}\right)\cos x + \text{sen}x.\overbrace{\text{sen}x.\csc x}^{1}}{\text{sen}^2x}$

$U = \dfrac{\text{sen}x + \text{sen}x}{\text{sen}^2x} = \dfrac{2\text{sen}x}{\text{sen}^2x} = \dfrac{2}{\text{sen}x}$

$\therefore U = 2\csc x$

Clave B

18. $A = 6(\text{sen}^4x + \cos^4x) - 4(\text{sen}^6x + \cos^6x)$

$A = 6(1 - 2\text{sen}^2x\cos^2x) - 4(1 - 3\text{sen}^2x\cos^2x)$

$A = 6 - 12\text{sen}^2x\cos^2x - 4 + 12\text{sen}^2x\cos^2x$

$A = 6 - 4 \qquad \therefore A = 2$

Clave B

19. $C = \dfrac{\sec x . \csc x - \text{sen}x - \cot x}{1 - \cos x}$

$$C = \dfrac{\dfrac{1}{\text{sen}x\cos x} - \text{sen}x - \dfrac{\cos x}{\text{sen}x}}{1 - \cos x}$$

$$C = \dfrac{\overbrace{1 - \cos^2 x}^{\text{sen}^2 x} - \text{sen}^2 x\cos x}{\text{sen}x . \cos x(1 - \cos x)}$$

$$C = \dfrac{\text{sen}^2 x(1 - \cos x)}{\text{sen}x . \cos x(1 - \cos x)} = \dfrac{\text{sen}x}{\cos x}$$

$\therefore C = \tan x$

Clave B

20. $A = \dfrac{\cos\theta(1 + \tan\theta)}{\text{sen}\theta(1 + \cot\theta)} = \dfrac{\cos\theta + \cos\theta . \tan\theta}{\text{sen}\theta + \text{sen}\theta . \cot\theta}$

$$A = \dfrac{\cos\theta + \cos\theta\left(\dfrac{\text{sen}\theta}{\cos\theta}\right)}{\text{sen}\theta + \text{sen}\theta\left(\dfrac{\cos\theta}{\text{sen}\theta}\right)} = \dfrac{\cos\theta + \text{sen}\theta}{\text{sen}\theta + \cos\theta} = 1$$

$\therefore A = 1$

Clave B

21. $A = \dfrac{\csc\theta - \cos\theta}{\sec\theta - \text{sen}\theta}$

$$A = \dfrac{\dfrac{1}{\text{sen}\theta} - \cos\theta}{\dfrac{1}{\cos\theta} - \text{sen}\theta} = \dfrac{\dfrac{1 - \text{sen}\theta\cos\theta}{\text{sen}\theta}}{\dfrac{1 - \text{sen}\theta\cos\theta}{\cos\theta}}$$

$$A = \dfrac{\cos\theta(1 - \text{sen}\theta\cos\theta)}{\text{sen}\theta(1 - \text{sen}\theta\cos\theta)} = \dfrac{\cos\theta}{\text{sen}\theta} = \cot\theta$$

$\therefore A = \cot\theta$

Clave B

Resolución de problemas

22. Dividimos las expresiones:

$A = \text{sen}x(\text{sen}x + \cos x - 1)$

$B = \sec x + \tan x(\cos x - 1) - 1$

$$\dfrac{A}{B} = \dfrac{\text{sen}^2 x + \text{sen}x(\cos x - 1)}{\sec x + \tan x(\cos x - 1) - 1}$$

$$\dfrac{A}{B} = \dfrac{1 - \cos^2 x + \text{sen}x(\cos x - 1)}{\dfrac{1}{\cos x} + \dfrac{\text{sen}x}{\cos x}(\cos x - 1) - 1}$$

$$\dfrac{A}{B} = \dfrac{(1 - \cos x)(1 + \cos x) - \text{sen}x(1 - \cos x)}{\dfrac{1 + \text{sen}x(\cos x - 1) - \cos x}{\cos x}}$$

$$\dfrac{A}{B} = \dfrac{(1 - \cos x)(1 + \cos x - \text{sen}x)}{(1 - \cos x)\left(\dfrac{1 - \text{sen}x}{\cos x}\right)}$$

$$\dfrac{A}{B} = \dfrac{\cos x^2 + \cos x(1 - \text{sen}x)}{1 - \text{sen}x}$$

$$\dfrac{A}{B} = \dfrac{(1 - \text{sen}x)(1 + \text{sen}x) + \cos x(1 - \text{sen}x)}{(1 - \text{sen}x)}$$

$$\dfrac{A}{B} = 1 + \text{sen}x + \cos x$$

Clave B

23. Multiplicamos R por $\cos^2 x$:

$$R = \left(\dfrac{\cos^2 x}{\cos^2 x}\right)\dfrac{\sec x^4 + \csc^4 x - \sec^4 x . \csc^4 x}{\csc^2 x}$$

$$R = \dfrac{\cos^2 x . \sec^4 x + \cos^2 x\csc^4 x - \cos^2 x\sec^4 x . \csc^4 x}{\cos^2 x . \csc^2 x}$$

$$R = \dfrac{\sec^2 x + \cos^2 x\csc^4 x - \sec^2 x\csc^4 x}{\cos^2 x . \csc^2 x}$$

$$R = \dfrac{1}{\cos^2 x}\left(\dfrac{\sec^2 x}{\csc x^2} + \dfrac{\cos^2 x\csc^4 x}{\csc^2 x} - \dfrac{\sec^2 x\csc^4 x}{\csc^2 x}\right)$$

$$R = \dfrac{1}{\cos^2 x}\left(\dfrac{\text{sen}^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\text{sen}^2 x} - \dfrac{1}{\text{sen}^2 x . \cos^2 x}\right)$$

$$R = \dfrac{1}{\cos^2 x}\left(\dfrac{\text{sen}^4 x + \cos^4 x - 1}{\text{sen}^2 x . \cos^2 x}\right)$$

$$R = \dfrac{1}{\cos^2 x}\left(\dfrac{-2\text{sen}^2 x\cos^2 x}{\text{sen}^2 x . \cos^2 x}\right)$$

$R = -2\sec^2 x$

Clave D

Nivel 3 (página 68) Unidad 4

Comunicación matemática

24. $M = \dfrac{1 - 2\text{sen}^2 x\cos^2 x + 3}{1 - 3\text{sen}^2 x\cos^2 x + 5}$

$$M = \dfrac{4 - 2\text{sen}^2 x\cos^2 x}{6 - 3\text{sen}^2 x . \cos^2 x} \Rightarrow M = \dfrac{2}{3}$$

$N = (\tan x . \cos x)^2 + (\cot x . \text{sen}x)^2$

$$N = \left(\dfrac{\text{sen}x}{\cos x} \cdot \cos x\right)^2 + \left(\dfrac{\cos x}{\text{sen}x} \cdot \text{sen}x\right)^2$$

$N = \text{sen}^2 x + \cos^2 x \Rightarrow N = 1$

$$\Rightarrow \dfrac{M}{N} = \dfrac{\dfrac{2}{3}}{1} = \dfrac{2}{3}$$

$\therefore 3M = 2N$

Clave E

25. Por teoría, tenemos:

I. $(1 + \text{sen}x + \cos x)^2 = 2(1 + \text{sen}x)(1 + \cos x)$

$\therefore A = 2$

II. $\tan x + \cot x = (\sec x . \csc x)$

$\therefore B = 1$

$\Rightarrow A + B = 2 + 1 = 3$

Clave B

Razonamiento y demostración

26. $L = \dfrac{(\tan x + 2\cot x)^2 + (2\tan x - \cot x)^2}{\tan^2 x + \cot^2 x}$

$$L = \dfrac{(\tan^2 x + 4\tan x\cot x + 4\cot^2 x) + (4\tan^2 x - 4\tan x\cot x + \cot^2 x)}{\tan^2 x + \cot^2 x}$$

$$L = \dfrac{5\tan^2 x + 5\cot^2 x}{\tan^2 x + \cot^2 x}$$

$$L = \dfrac{5(\tan^2 x + \cot^2 x)}{(\tan^2 x + \cot^2 x)}$$

$\therefore L = 5$

Clave B

27. $U = \dfrac{\sec^2 x . \csc^2 x - \tan^2 x - \cot^2 x}{\underbrace{\text{sen}^2 x + \cos^2 x}_{1}}$

$U = \sec^2 x . \csc^2 x - \tan^2 x - \cot^2 x$

Por identidad auxiliar:

$\sec^2 x . \csc^2 x = \sec^2 x + \csc^2 x$

Entonces:

$U = \sec^2 x + \csc^2 x - \tan^2 x - \cot^2 x$

$U = \underbrace{\sec^2 x - \tan^2 x}_{1} + \underbrace{\csc^2 x - \cot^2 x}_{1}$

$\therefore U = 1 + 1 = 2$

Clave B

28. $D = \dfrac{\cos^2 x . \sec x + 2\cot x . \text{sen}x}{1 - \text{sen}^2 x}$

$$D = \dfrac{\overbrace{\cos x . \cos x . \sec x}^{1} + 2\left(\dfrac{\cos x}{\text{sen}x}\right) . \text{sen}x}{\cos^2 x}$$

$$D = \dfrac{\cos x + 2\cos x}{\cos^2 x} = \dfrac{3\cos x}{\cos^2 x} = \dfrac{3}{\cos x}$$

$\therefore D = 3\sec x$

Clave D

29. $A = \dfrac{\sec^2 x\csc^2 x - \sec^2 x - 1}{\cot x}$

$$A = \dfrac{\sec^2 x(\csc^2 x - 1) - 1}{\cot x} = \dfrac{\sec^2 x(\cot^2 x) - 1}{\cot x}$$

$$A = \dfrac{\dfrac{1}{\cos^2 x}\left(\dfrac{\cos^2 x}{\text{sen}^2 x}\right) - 1}{\cot x} = \dfrac{\dfrac{1}{\text{sen}^2 x} - 1}{\cot x}$$

$$A = \dfrac{\csc^2 x - 1}{\cot x} = \dfrac{\cot^2 x}{\cot x} = \cot x$$

$\therefore A = \cot x$

Clave C

30. $U = \dfrac{(\text{sen}x + \cos x)^2 + 4\tan x\cos^2 x - 1}{(\text{sen}x - \cos x)^2 + 4\cot x\,\text{sen}^2 x - 1}$

$$U = \dfrac{\text{sen}^2 x + 2\text{sen}x\cos x + \cos^2 x + 4\dfrac{\text{sen}x}{\cos x} . \cos^2 x - 1}{\text{sen}^2 x - 2\text{sen}x\cos x + \cos^2 x + 4\dfrac{\cos x}{\text{sen}x} . \text{sen}^2 x - 1}$$

$$U = \dfrac{\overbrace{\text{sen}^2 x + \cos^2 x}^{1} + 2\text{sen}x\cos x + 4\text{sen}x\cos x - 1}{\underbrace{\text{sen}^2 x + \cos^2 x}_{1} - 2\text{sen}x\cos x + 4\cos x\,\text{sen}x - 1}$$

$$U = \dfrac{1 + 6\text{sen}x\cos x - 1}{1 + 2\text{sen}x\cos x - 1} = \dfrac{6\text{sen}x\cos x}{2\text{sen}x\cos x}$$

$\therefore U = 3$

Clave C

31. $A = \dfrac{\text{sen}\theta + \cot\theta}{\csc\theta + \tan\theta} = \dfrac{\text{sen}\theta + \dfrac{\cos\theta}{\text{sen}\theta}}{\dfrac{1}{\text{sen}\theta} + \dfrac{\text{sen}\theta}{\cos\theta}}$

$$A = \dfrac{\dfrac{\text{sen}^2\theta + \cos\theta}{\text{sen}\theta}}{\dfrac{\cos\theta + \text{sen}^2\theta}{\text{sen}\theta . \cos\theta}}$$

$$A=\frac{\operatorname{sen}\theta.\cos\theta(\operatorname{sen}^2\theta+\cos\theta)}{\operatorname{sen}\theta(\cos\theta+\operatorname{sen}^2\theta)}$$

$\therefore A=\cos\theta$

Clave C

32. $$L=\frac{(\operatorname{sen}x+\cos x+1)(\operatorname{sen}x+\cos x-1)}{\operatorname{sen}x.\cos x}$$

$$L=\frac{(\operatorname{sen}x+\cos x)^2-1^2}{\operatorname{sen}x.\cos x}$$

$$L=\frac{\operatorname{sen}^2x+2\operatorname{sen}x\cos x+\cos^2x-1}{\operatorname{sen}x.\cos x}$$

$$L=\frac{\overbrace{\operatorname{sen}^2x+\cos^2x}^{1}-1+2\operatorname{sen}x\cos x}{\operatorname{sen}x\cos x}$$

$$L=\frac{1-1+2\operatorname{sen}x\cos x}{\operatorname{sen}x\cos x}$$

$$L=\frac{2\operatorname{sen}x\cos x}{\operatorname{sen}x\cos x}=2$$

$\therefore L=2$

Clave A

33. $$A=\frac{\sec x.\csc x-\cot x}{\operatorname{sen}x}$$

$$A=\frac{\frac{1}{\cos x\operatorname{sen}x}-\frac{\cos x}{\operatorname{sen}x}}{\operatorname{sen}x}=\frac{\frac{(1-\cos^2x)}{\operatorname{sen}x\cos x}}{\operatorname{sen}x}$$

$$A=\frac{\operatorname{sen}^2x}{\operatorname{sen}^2x.\cos x}=\frac{1}{\cos x}=\sec x$$

$\therefore A=\sec x$

Clave D

Resolución de problemas

34. Hallamos la expresión M:

$$M=\frac{\operatorname{sen}^4x+\operatorname{sen}^2x\cos^2x+\cos^2x}{1-\cos^2x}$$

$$M=\frac{\operatorname{sen}^4x+\operatorname{sen}^2x\cos^2x+\cos^2x}{\operatorname{sen}^2x}$$

$$M=\frac{\operatorname{sen}^4x}{\operatorname{sen}^2x}+\frac{\operatorname{sen}^2x\cos^2x}{\operatorname{sen}^2x}+\frac{\cos^2x}{\operatorname{sen}^2x}$$

$M=\operatorname{sen}^2x+\cos^2x+\cot^2x$
$M=1+\cot^2x$

- Reemplazamos en C.

$$C=\sqrt{\frac{\operatorname{sen}^2x+\csc^2x}{1+\cot^2x}};\ x\in IC$$

$$C=\sqrt{\frac{\sec^2x+\csc^2x}{\csc^2x}};\ x\in IC$$

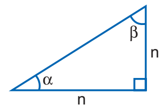
$$C=\sqrt{\frac{\sec^2x}{\csc^2x}+\frac{\csc^2x}{\csc^2x}};\ x\in IC$$

$$C=\sqrt{1+\tan^2x}=\sqrt{\sec^2x}=\sec x$$

$\therefore C=\sec x$

Clave C

35. Tenemos un triángulo rectángulo



$\Rightarrow \tan\alpha=\cot\beta$

Reemplazamos en k:

$k=(1+\tan\alpha)(1+\tan\beta)-2$

$k=(1+\cot\beta)(1+\tan\beta)-2$

$$k=\left(1+\frac{\cos\beta}{\operatorname{sen}\beta}\right)\left(1+\frac{\operatorname{sen}\beta}{\cos\beta}\right)-2$$

$$k=\frac{(\operatorname{sen}\beta+\cos\beta)(\cos\beta+\operatorname{sen}\beta)}{\operatorname{sen}\beta.\cos\beta}-2$$

$$k=\frac{\operatorname{sen}^2\beta+\cos^2\beta+2\operatorname{sen}\beta\cos\beta}{\operatorname{sen}\beta.\cos\beta}-2$$

$$k=\frac{(1+2\operatorname{sen}\beta.\cos\beta)}{\operatorname{sen}\beta.\cos\beta}-2$$

$$k=\frac{1}{\operatorname{sen}\beta.\cos\beta}+\frac{2\operatorname{sen}\beta.\cos\beta}{\operatorname{sen}\beta.\cos\beta}-2$$

$k=\csc\beta.\sec\beta+2-2$

$k=\csc\beta.\sec\beta=\sec\alpha.\sec\beta$

Clave B

APLICAMOS LO APRENDIDO (página 70) Unidad 4

1. Hallamos la equivalencia:

$1hm^3 = 1(100\ m)^3 = 10^6 m^3$

Desarrollamos:

$1\ hm^3$ ——— $10^6\ m^3$

x ——— $3500\ m^3$

$\Rightarrow x = \dfrac{3500m^3 .\ 1hm^3}{10^6 m^3}$ $\quad \therefore x = 0{,}0035\ hm^3$

Clave B

2. Hallamos la equivalencia:

$x = 0{,}001\ hm^2 = 0{,}001(10^4\ cm)^2 = 0{,}001 \times 10^8\ cm^2$

$\therefore x = 1 \times 10^5\ cm^2$

Clave D

3. Hallamos la equivalencia:

$x = 0{,}33\ dag = 0{,}33(10^2\ dg) = 33\ dg$

$\therefore x = 33\ dg$

Clave C

4. $a + x = 0{,}8\ m = 80\ cm$

$x + b = 560\ mm = 56\ cm$

$2x + a + b = 136\ cm$

$2x + 0{,}0066\ hm = 136\ cm$

$2x + 66\ cm = 136\ cm$

$2x = 70\ cm \Rightarrow x = 35\ cm$

Clave E

5. El auto A: $7{,}5\ l$ ——— $100\ km$

x ——— $265\ km$

$\Rightarrow x = \dfrac{265(7{,}5\ l)}{100} = 19{,}875\ l$

El auto B: $0{,}082\ hl$ ——— $100\ km$

y ——— $265\ km$

$\Rightarrow y = \dfrac{0{,}082 \times 10^2 l(265)}{100} = 21{,}73\ l$

Total consumido:

$T = 19{,}875\ l + 21{,}73\ l = 41{,}605\ l$

Clave E

6. Hallamos la cantidad de combustible usado:

$2 \times 21{,}73\ l = 43{,}46\ l$

Gastaremos:

P = (S/.15)(43,46)

P = S/.651,9

Clave A

7. 0,5 kg – S/.0,60 ⇒ 1 kg – S/.1,20 ⇒ 1000 g P ⇒ S/.0,48 ⇒ 400 g

30 dag – S/.1,50 ⇒ S/.1,50 ⇒ 300 g A ⇒ S/.7,5 ⇒ 1500 g

0,8 mag – S/.12,00 ⇒ S/.12,00 ⇒ 8000 g Z ⇒ S/.0,3 ⇒ 200 g

1200 g – S/.2,40 ⇒ S/.2,40 ⇒ 1200 g F ⇒ S/.0,8 ⇒ 400 g

Hallamos la suma total:

S = S/.0,48 + S/.7,5 + S/.0,3 + S/.0,8 = S/.9,08

Clave B

8. 30 dag ⇒ S/.1,50

30(0,01 kg) ⇒ S/.1,50

$\Rightarrow x = \dfrac{S/.18{,}00 \times (30 \times 0{,}01\,kg)}{S/.1{,}50}$

x ⇒ S/.18,00 $\quad \therefore x = 3{,}6\ kg$

Clave D

9. Convertimos todos los volúmenes a mm^3

Lunes $\Rightarrow 10^{-3}\ dm^3 = 10^{-3}(10^2\ mm)^3 = 1 \times 10^3\ mm^3$

Martes $\Rightarrow 10^{-6}\ m^3 = 10^{-6}(10^3\ mm)^3 = 1 \times 10^3\ mm^3$

Miércoles $\Rightarrow 1200\ mm^3$

Jueves $\Rightarrow 750\ cm^3 = 750(10\ mm)^3 = 75 \times 10^4\ mm^3$

Viernes $\Rightarrow 10^{-11}\ hm^3 = 10^{-11}(10^5\ mm)^3 = 10^4\ mm^3$

Sábado $\Rightarrow 800\ mm^3$

Domingo $\Rightarrow 10^{-8}\ dam^3 = 10^{-8}(10^4\ mm)^3 = 10^4\ mm^3$

∴ El día de mayor consumo es jueves.

Clave D

10. 3 dormitorios $\Rightarrow 3 \times 500\ 000\ cm^2 = 3 \times 50\ m^2 = 150\ m^2$

1 comedor $\Rightarrow 0{,}3\ dam^2 = 0{,}3 \times 10^2\ m^2 = 30\ m^2$

1 cocina $\Rightarrow 0{,}006\ hm^2 = 0{,}006 \times 10^4\ m^2 = 60\ m^2$

1 cochera $\Rightarrow 80 \times 10^{-6}\ km^2 = 80 \times 10^{-6} \times 10^6\ m^2 = 80\ m^2$

2 salas $\Rightarrow 2 \times 4500\ dm^2 = 9000 \times 10^{-2}\ m^2 = 90\ m^2$

Hallamos el área total:

$A_T = 150\ m^2 + 30\ m^2 + 60\ m^2 + 80\ m^2 + 90\ m^2$

$A_T = 410\ m^2$

Clave C

11. Hallamos los metros que abastece 1 balde:

x = 200 hm + 800 dam + 12 000 m

x = 20 000 m + 8000 m + 12 000 m

x = 40 000 m

Dividimos: n.º baldes $= \dfrac{1200 \times 10^3 m}{40000m} = 30$ baldes

Clave A

12. Hallamos la capacidad total a llenar:

$C_T = 0{,}48\ mal + 3 \times 10^6\ cl$

$C_T = 4800\ l + 3000\ l = 7800\ l$

Hallamos el tiempo:

12 dal ⇒ 1 min

120 l ⇒ 1 min

7800 l ⇒ x

$\Rightarrow x = \dfrac{7800\ l \times 1\ min}{120\ l}$

x = 65 min $\quad \therefore x = 1\ h\ 5\ min$

Clave E

13. 5000 dag < x < 650 hg

50 kg < x < 65 kg

∴ x = 18

Clave C

14. Hallamos el valor de k:

k = 300 g + 548 dag + 432,2 hg + 6 kg

k = 0,3 kg + 5,48 kg + 43,22 kg + 6 kg

k = 55,00 kg

El número de estudiantes:

x < 55 kg

∴ n.º de estudiantes = 4 + 10 + 5 = 19

Clave A

PRACTIQUEMOS

Nivel 1 (página 72) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $2{,}7\ hm = 2{,}7(10^2\ m) = 270\ m$

$34{,}6\ dam = 34{,}6(10\ m) = 346\ m$

$x = 270\ m + 346\ m = 616\ m$

Clave A

4. $800\ dm = 800(10^{-2}\ dam) = 8\ dam$

$0{,}2\ km = 0{,}2(10^2\ dam) = 20\ dam$

$x + 8\ dam = 20\ dam$

$\therefore x = 12\ dam$

Clavo E

5. $0{,}3\ l = 0{,}3(10^2\ cl) = 30\ cl$

$10^{-5}\ hl = 10^{-5}(10^4\ cl) = 0{,}1\ cl$

$x = 30\ cl + 0{,}1\ cl = 30{,}1\ cl$

Clave C

6. $900\ dal = 900(10^{-2}\ kl) = 9\ kl$

$x + 9\ kl = 12\ kl$

$\Rightarrow x = 3\ kl = 3000\ l$

Clave B

7. $0{,}02\ hg = 0{,}02(10^4\ cg) = 200\ cg$

$300\ mg = 300\ (10^{-1}\ cg) = 30\ cg$

$x = 200\ cg - 30\ cg = 170\ cg$

Clave D

8. $0{,}0003\ kg = 0{,}0003(10^4\ dg) = 3\ dg$

$0{,}2\ hg = 0{,}2(10^3\ dg) = 200\ dg$

$x + 3\ dg = 200\ dg \Rightarrow x = 197\ dg$

Clave A

9. $0{,}000004\ hm^3 = 4 \times 10^{-6}(10^3\ dm)^3 = 4 \times 10^3\ dm^3$

$1\ m^3 = 1 \times 10^3\ dm^3$

$x = 4 \times 10^3\ dm^3 - 1 \times 10^3\ dm^3 = 3 \times 10^3\ dm^3$

$\therefore x = 3000\ dm^3$

Clave C

10. $0{,}004\ m^2 = 4 \times 10^{-3}(10^2\ cm)^2 = 40\ cm^2$

$1000\ mm^2 = 10^3(10^{-2}\ cm^2) = 10\ cm^2$

$40\ cm^2 - x = 10\ cm^2 \Rightarrow x = 30\ cm^2$

Clave D

Resolución de problemas

11. De la moto: 90 hm – 2 horas

45 hm – 1 hora

4,5 km – 1 hora

Del auto: 1800 dam – 1 hora

18 km – 1 hora

Cuando el auto alcanza a la moto han recorrido la misma distancia, entonces:

$(4,5\ \text{km})(x + 4) = (18\ \text{km})(x)$

$4,5x + 18 = 18x$

$18 = 13,5x$

$x = 1,3$ horas

Clave E

12. $1,8\ \text{kl} = 500\ \text{l} + 400\ \text{l} + x$

$1800\ \text{l} = 900\ \text{l} + x$

$\Rightarrow x = 900\ \text{l}$

Dividimos entre la capacidad de cada botella.

$\Rightarrow$ n.º botellas $= \dfrac{900\ \text{l}}{1,5\ \text{l}} = 600$

Clave A

Nivel 2 (página 72) Unidad 4

Comunicación matemática

13.

14.

Razonamiento y demostración

15. $2\ \text{dam} \Rightarrow 2(10^3\ \text{cm}) = 2000\ \text{cm}$

$36\ \text{m} \Rightarrow 36(10^2\ \text{cm}) = 3600\ \text{cm}$

$56\ \text{dm} \Rightarrow 56(10\ \text{cm}) = 560\ \text{cm}$

Hallamos la altura total

$h = 2000\ \text{cm} + 3600\ \text{cm} + 560\ \text{cm}$

$\therefore h = 6160\ \text{cm}$

Clave A

16. $40\ \text{hl} \Rightarrow 40(10^3\ \text{dl}) = 40\,000\ \text{dl}$

$365\ \text{dal} \Rightarrow 365(10^2\ \text{dl}) = 36\,500\ \text{dl}$

$250\ \text{l} \Rightarrow 250(10\ \text{dl}) = 2500\ \text{dl}$

La capacidad total es:

$C = 40\,000 + 36\,500 + 2500$

$C = 79\,000\ \text{dl}$

$\therefore C = 79 \times 10^3\ \text{dl}$

Clave B

17. $x\ \text{mag} = 2800\text{hg} + 36 \times 10^3\text{dag} + 4,28 \times 10^6\text{g}$

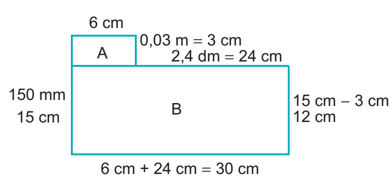
$x\ \text{mag} = 2800(10^{-2}\text{mag}) + 36 \times 10^3(10^{-3}\text{mag}) + 4,28 \times 10^6(10^{-4}\ \text{mag})$

$x\ \text{mag} = 28\ \text{mag} + 36\ \text{mag} + 428\ \text{mag}$

$x\ \text{mag} = 492\ \text{mag}$

Clave C

18.



$A_T = A + B$

$A_T = (3\ \text{cm})(6\ \text{cm}) + (30\ \text{cm})(12\ \text{cm})$

$A_T = 18\ \text{cm}^2 + 360\ \text{cm}^2$

$A_T = 378\ \text{cm}^2$

Clave D

19. Aplicamos teorema de Pitágoras:

$(AB)^2 + (BC)^2 = (AC)^2$

$x^2 + (0,04\ \text{dm})^2 = (0,005\ \text{m})^2$

$x^2 + (4\ \text{mm})^2 = (5\ \text{mm})^2$

$x^2 = 25\ \text{mm}^2 - 16\ \text{mm}^2$

$x^2 = 9\ \text{mm}^2 \Rightarrow x = 3\ \text{mm}$

El área del triángulo:

$\dfrac{b.h}{2} = \dfrac{(4\ \text{mm})(3\ \text{mm})}{2} = 6\ \text{mm}^2$

Clave A

20. $C_T = 1080\ \text{l} + 42,6\ \text{hl} + 37,5\ \text{kl}$

$C_T = 108\ \text{dal} + 426\ \text{dal} + 3750\ \text{dal}$

$C_T = 4284\ \text{dal}$

Clave C

Resolución de problemas

21. Transformamos la lista en una sola unidad de masa(gramo):

Plátano	0,05 mag = 500 g × S/.2,80
Manzana	3000 dg = 300 g × S/.1,20
Papaya	400 dag = 4000 g × S/.6,40
Naranja	6 hg = 600 g × S/.2,40

Transformamos la lista de campos:

plátano $\longrightarrow$ 5000 g

manzana $\longrightarrow$ 4 hg = 400 g

papaya $\longrightarrow$ 0,1 mag = 1000 g

naranja $\longrightarrow$ 2 kg = 2000 g

Calculamos el gasto total :

plátano 5000 g $\Rightarrow$ 10 (S/.2,80) = S/.28,00

manzana 400 g $\Rightarrow \dfrac{\text{S/.}\,1,20}{300\ \text{g}} \times (400\ \text{g}) = \text{S/.}1,60$

papaya 1000 g $\Rightarrow \dfrac{\text{S/.}\,6,40}{4000\ \text{g}} \times (1000\ \text{g}) = \text{S/.}1,60$

naranja 2000 g $\Rightarrow \dfrac{\text{S/.}2,40}{600\ \text{g}}\ (2000\ \text{g}) = \text{S/.}8,00$

$P_T = \text{S/.}28,00 + \text{S/.}1,60 + \text{S/.}1,60 + \text{S/.}8,00$

$P_T = \text{S/.}39,20$

Clave B

22. Tenemos en cuenta:

manzana 300 g × S/.1,20

papaya 4000 g × S/.6,40

Supongamos que se compró

manzana $\quad A\ \text{g}$

papaya $\quad B\ \text{g}$ $\quad \Rightarrow \dfrac{A}{B} = \dfrac{2}{1} = k$

$A = 2k$

$B = k$

Tenemos:

$\dfrac{\text{S/.}1,20}{300\text{g}}(A\ \text{g}) + \dfrac{\text{S/.}6,40}{4000\text{g}}(B\ \text{g}) = \text{S/.}14,40$

$\dfrac{\text{S/.}1,20(2k)}{300\ \text{g}} + \dfrac{\text{S/.}6,40(k)}{4000\ \text{g}} = \text{S/.}14,40$

$\dfrac{96k + 19,2k}{120} = 1440$

$115,2\ (k) = 120(1440)$

$k = 1500 \Rightarrow A = 2k$

$\Rightarrow A = 2(1500\ \text{g}) = 3000\ \text{g}$

$\therefore A = 3000\ \text{g} = 3000(10^{-2}\ \text{hg}) = 30\ \text{hg}$

Clave E

Nivel 3 (página 73) Unidad 4

Comunicación matemática

23. $M = \sqrt{0,3\ \text{dm} \times 4\ \text{cm} + 0,02\ \text{m} \times 20\ \text{mm}}$

$M = \sqrt{3\ \text{cm} \times 4\ \text{cm} + 2\ \text{cm} \times 2\ \text{cm}}$

$\therefore M = 4\ \text{cm}$

$N = \dfrac{0,022\ \text{m} \times 20\ \text{mm} + 0,009\ \text{m} \times 0,4\ \text{dm}}{4 \times 0,0005\ \text{dam}}$

$N = \dfrac{2,2\ \text{cm} \times 2\ \text{cm} + 0,9\ \text{cm} \times 4\ \text{cm}}{0,002\ \text{dam}}$

$N = \dfrac{4,4\ \text{cm}^2 + 3,6\ \text{cm}^2}{2\ \text{cm}} = \dfrac{8\ \text{cm}^2}{2\ \text{cm}}$

$\therefore N = 4\ \text{cm}$

$M = N$

Clave C

24.

I. $10^{-2}\ \text{dm} = 10^{-2}(10^2\ \text{mm}) = 1\ \text{mm} \neq 10\ \text{mm}$ F

II. $10^4\ \text{cl} = 10^4(10^{-6}\text{mal}) = 10^{-2}\ \text{mal} = 10^{-2}\ \text{mal}$ V

III. $10^{-1}\ \text{dag} = 10^{-1}(10^3\ \text{cg}) = 10^2\ \text{cg} = 10^2\ \text{cg}$ V

IV. $10^6\ \text{cm}^3 = (10^2\ \text{cm})^3 = 1\ \text{m}^3 \neq 10\ \text{m}^3$ F

V. $10^{-4}\ \text{dam}^2 = 10^{-4}(10^3\ \text{cm})^2 = 10^2\ \text{cm}^2 = 10^2\ \text{cm}^2$ V

$\therefore$ I y IV son falsas

Clave E

Razonamiento y demostración

25. $AB = AC - BC$

$2x = 0,04\ \text{dam} - 80\ \text{mm}$

$2x = 40\ \text{cm} - 8\ \text{cm} = 32\ \text{cm}$

$2x = 32\ \text{cm} \Rightarrow x = 16\ \text{cm}$

$\therefore x = 0,16\ \text{m}$

Clave B

26. Hallamos su capacidad total

$C_t = 12\ \text{dal} + 480\ \text{l} + 0,32 \times 10^4\ \text{cl}$

$C_t = 120\ \text{l} + 480\ \text{l} + 32\ \text{l}$

$C_t = 632\ l$

$C_t = C_{llena} + C_{vacía}$

$632\ l = (632\ l)\left(\frac{3}{4}\right) + C_{vacía}$

$(632\ l)\left(1 - \frac{3}{4}\right) = C_{vacía}$

$(632\ l)\left(\frac{1}{4}\right) = C_{vacía} \qquad \therefore\ C_{vacía} = 158\ l$

Clave A

27. $x = 7500\ dg + 250\ g + 0{,}3\ kg$

$x = 0{,}75\ kg + 0{,}25\ kg + 0{,}3\ kg$

$x = 1{,}3\ kg$

Clave D

28. Contamos 90 cubitos

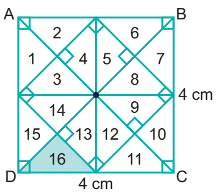
$\Rightarrow\ V = 90\ (10\ dm^3)$

$V = 900\ dm^3$

$V = 0{,}9\ m^3$

Clave E

29. De la figura:



Notamos que todos los pequeños triángulos poseen igual área:

$\Rightarrow\ A_S = \frac{A_{TOTAL}}{16}$

$A_S = \frac{4\ cm(4\ cm)}{16} = \frac{16\ cm^2}{16}$

$\Rightarrow\ A_S = 1\ cm^2 \qquad \therefore\ A_S = 10^{-2}\ dm^2$

Clave B

30. $A = 0{,}33\ kl = 3{,}3\ hl$

$B = 1200\ dl = 1{,}2\ hl$

$C = 180\ l = 1{,}8\ hl$

$D = ?\ = \frac{1}{8}$ (barril)

$A + B + C = \frac{7}{8}$ (barril)

$3{,}3\ hl + 1{,}2\ hl + 1{,}8\ hl = \frac{7}{8}$ (barril)

$6{,}3\ hl = \frac{7}{8}$ barril

$7{,}2\ hl =$ barril

$\Rightarrow\ D = \frac{1}{8}$ (barril) $= \frac{1}{8}(7{,}2\ hl) \qquad \therefore\ 0{,}9\ hl$

Clave A

Resolución de problemas

31.

Rutas	distancia	
2.°	1700 hm	= 170 km +
4.°	4×10^5 m	= 400 km
6.°	650 km	= 650 km

$R_1 = 1220$ km

Rutas	distancia	
1.°	250 km	= 250 km +
4.°	4×10^5 m	= 400 km
5.°	72 mam	= 720 km

$R_2 = 1370$ km

$R_2 - R_1 = 1370\ km - 1220\ km$

$\therefore\ R_2 - R_1 = 150\ km$

Clave C

32. Oferta A:

$A \times L = 40\ m \times 75\ m = 3000\ m^2$

$A = 3000$ (S/.5) = S/.15 000

Oferta B:

$A \times L = 400\ dm \times 750\ dm = 3 \times 10^5\ dm^2$

$B = 3 \times 10^5\ dm^2$ (S/.0,02) = S/.6000

Oferta C:

$A \times L = 0{,}4\ hm \times 0{,}75\ hm = 0{,}3\ hm^2$

$C = 0{,}3$ (S/.36 000) = S/.10 800

Ordenamos:

$A > C > B$

Clave A

MARATÓN MATEMÁTICA (página 75)

1. Tenemos:

$M = \tan^2 x + \frac{\cos x}{\cos x - \sec y} = \tan^2 x + \frac{\cos x}{\cos x - \frac{1}{\cos x}}$

$\Rightarrow\ \frac{\cos x}{\frac{\cos^2 x - 1}{\cos x}} = \frac{\cos^2 x}{-\operatorname{sen}^2 x} + \tan^2 x$

$M = -\tan^2 x + \tan^2 x \qquad \therefore\ M = 0$

Clave C

2. De la condición tenemos:

$\csc^2 x + \operatorname{sen}^2 x = 3$

$\csc^2 x - 2\operatorname{sen}x\csc x + \operatorname{sen}^2 x = 3 - 2$

$(\csc x - \operatorname{sen}x)^2 = 1$

$R^2 = 1$

$R = \pm 1 \qquad \therefore\ R = 1$

Clave A

3. De la condición:

$\cos\theta - \cot\theta = 1$

$\cos\theta = 1 + \cot\theta$

$\cos\theta = 1 + \frac{\cos\theta}{\operatorname{sen}\theta} = \cos\theta = \frac{\operatorname{sen}\theta + \cos\theta}{\operatorname{sen}\theta}$

$\operatorname{sen}\theta\cos\theta = \operatorname{sen}\theta + \cos\theta$

Dividimos entre $\cos\theta$:

$\frac{\operatorname{sen}\theta\cos\theta}{\cos\theta} = \frac{\operatorname{sen}\theta + \cos\theta}{\cos\theta}$

$\operatorname{sen}\theta = \tan\theta + 1$

$-1 = \tan\theta - \operatorname{sen}\theta$

Clave A

4. Sabemos:

$RT\left(\frac{\pi}{2} + \alpha\right) = (\text{signo})\text{Co } RT(\alpha)$

Entonces

$\cos 91° = \cos(90° + 1°) = -\operatorname{sen}1°$

$\cos 92° = \cos(90° + 2°) = -\operatorname{sen}2°$

$\vdots \qquad \vdots \qquad \vdots$

$\cos 95° = \cos(90° + 5°) = -\operatorname{sen}5°$

Reemplazamos en la expresión:

$M = \frac{\cos 91° + \cos 92° + ... + \cos 95°}{\operatorname{sen}1° + \operatorname{sen}2° + ... + \operatorname{sen}5°}$

$M = \frac{(-\operatorname{sen}1°) + (-\operatorname{sen}2°) + ... + (-\operatorname{sen}5°)}{\operatorname{sen}1° + \operatorname{sen}2° + ... + \operatorname{sen}5°}$

$M = \frac{-(\operatorname{sen}1° + \operatorname{sen}2° + ... + \operatorname{sen}5°)}{(\operatorname{sen}1° + \operatorname{sen}2° + ... + \operatorname{sen}5°)}$

$\therefore\ M = -1$

Clave D

5. Sabemos:

$RT\left(\frac{\pi}{2} + \theta\right) = (\text{signo})\ .\ \text{Co } RT(\theta)$

$RT(2\pi + \theta) = RT(\theta)$

Entonces:

$R = \sec\left(\frac{\pi}{2} + \theta\right) + \csc(2\pi + \theta) - \csc\theta + \csc\theta = 0$

$\therefore\ R = 0$

Clave E

6. Tenemos:

$\operatorname{sen}315° = \operatorname{sen}(360° - 45°) = -\operatorname{sen}45° = \frac{-\sqrt{2}}{2}$

$\cos 397° = \cos(360° + 37°) = \cos 37° = \frac{4}{5}$

Reemplazamos en P:

$P = \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) + 5\left(\frac{4}{5}\right)$

$P = -1 + 4 = 3 \qquad \therefore\ P = 3$

Clave B

7. Tenemos:

$\operatorname{sen}(5\pi - \theta) = \operatorname{sen}(4\pi + \pi - \theta) = \operatorname{sen}(\pi - \theta)$
$= \operatorname{sen}\theta$

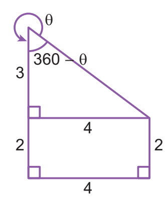
$\operatorname{sen}(7\pi + \theta) = \operatorname{sen}(6\pi + \pi + \theta) = \operatorname{sen}(\pi + \theta)$
$= -\operatorname{sen}\theta$

Reemplazamos en k:

$k = \operatorname{sen}\theta - \operatorname{sen}\theta = 0 \qquad \therefore\ k = 0$

Clave B

8. Del gráfico:



Tenemos:

$\tan(360 - \theta) = \frac{4}{3}$

$-\tan\theta = \frac{4}{3}$

$\tan\theta = -\frac{4}{3}$

Clave E

9. Tenemos:

$\operatorname{sen}127° = \operatorname{sen}(180° - 53°) = \operatorname{sen}53°$

$\tan 151° = \tan(180° - 29°) = -\tan 29°$

$\tan 209° = \tan(180° + 29°) = \tan 29°$

Reemplazamos:

$\operatorname{sen}\theta = \frac{\operatorname{sen}53°(-\tan 29°)}{(\tan 29°)}$

$\operatorname{sen}\theta = -\operatorname{sen}53°$

$\therefore\ \theta = -53°$

Clave C